String Free Energy, Hagedorn

and

Gauge/String Duality

Oleg Andreev

TU, München & Landau, Moscow

Based on O. Andreev, JHEP 0903, 098 (2009).

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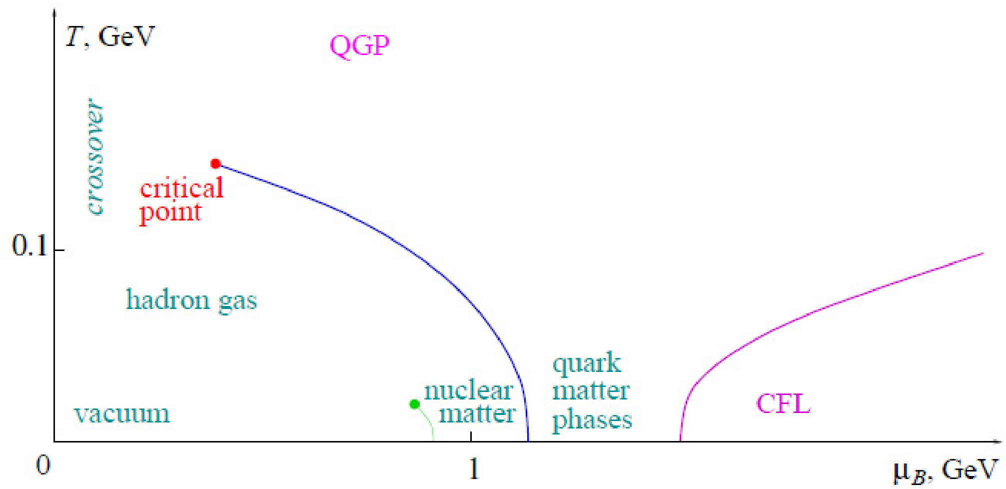


The QCD phase diagram
(from a review of Stephanov, hep-lat/0701002)

At zero chemical potential, the finite-temperature QCD transition is not a real phase transition, but an analytic crossover !

*** Y. Aoki, G. Endrodi, Z. Fodor, S.D. Katz, and K.K. Szabo, Nature 443, 6675 (2006).**

The string free energy is proportional to

$$\int^{\infty} [dh]\, \mathrm{e}^{4\pi h} \mathrm{e}^{-h} /\sqrt{\alpha'} T$$

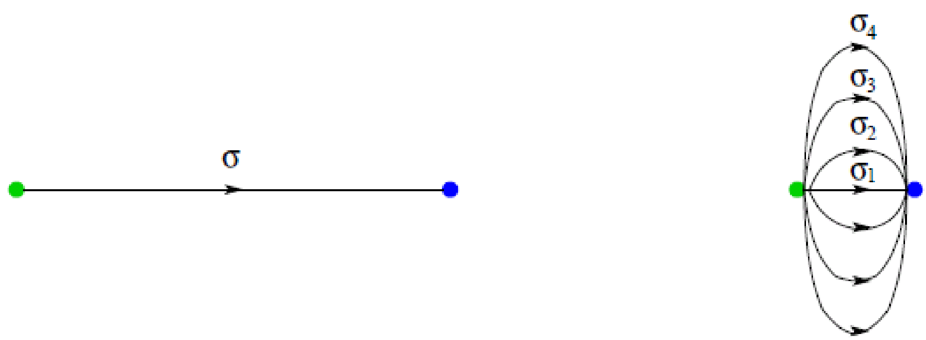
It diverges for temperatures greater than the Hagedorn temperature

$$T_{\mathrm{H}} = 1/4\pi\sqrt{\alpha'}$$

*** R. Hagedorn, Nuovo Cim. Suppl. 3, 147 (1965):**
Yu.B. Rumer, Zh.Eksp.Teor.Fiz. 8, 1899 (1960).

The problem: if strings are indeed relevant for QCD then one has to show that a stringy description is also valid for high T.

Flux Tube Picture



Faraday's lines of force

In the case of continuous spectrum of string tensions it seems natural to promote σ to a new spacetime coordinate. If so, then strings effectively live in some warped 5-dimensional space with σ playing a role of the fifth dimension.

Another possibility is a discrete spectrum of string tensions. Or even both discrete and continuous pieces.

A "fat string" of lattice QCD is a collection of thin strings with different tensions.

Quantized String Tension

Take an ensemble of strings whose tensions take discrete values (quantized). Andreev and Siegel (2005).

The simplest possible proposal for the free energy of such an ensemble seems to be that F is a sum of string free energies

$$F = \sum_{n=1}^{\infty} w_n F(n)$$

Now we have a set of Hagedorn temperatures. Its range is from the original one (with n=1) to infinity (with infinite n)

For higher states it leads to

$$\sum_n w_n \int^{\infty} [dh] \, \mathrm{e}^{4\pi h} \mathrm{e}^{-h\sqrt{\mathcal{P}_k}/\sqrt{\alpha'}T}$$

Here

$\alpha'_n = \alpha'/\mathcal{P}_k(n)$

$\mathcal{P}_k(n)$ **is a polynomial of degree k.**

The integrals are convergent for

$$n_* < n \, ,$$

with the low bound defined from

$$\mathcal{P}_k(n) = T^2/T_{\mathrm{H}}^2 \, ,$$

The refined proposal is

$$F = \sum_{\max\{1,1+[n_*]\}}^{\infty} w_n F(n)\,,$$

The physical meaning is that in the ensemble we have to keep only strings whose Hagedorn temperatures are above T.

If the weight factor shows a power law behavior for large n, then F also behaves as a power of T. In particular, the same factor as used in Andreev and Siegel (2006) for the hard scattering limit results in the free energy density

$$f \sim T^4$$

Warped Geometry

Quite similar to Polchinski and Strassler (2002) for the hard scattering limit.

Take the Schwarzschild black hole in AdS

$$ds^2 = \frac{r^2}{R^2}(l dt^2 + d\vec{x}^2) + \frac{R^2}{r^2} l^{-1} dr^2 + ds_{\rm X}^2 \,, \quad l = 1 - \frac{r_{\rm T}^4}{r^4} \,, \quad r_{\rm T} = \pi R^2 T$$

In the large-r region, we have
(using only the zero mode for a world-sheet field r)

$$f = \frac{F}{V} = \frac{1}{\alpha'^5} \frac{V_{\rm X}}{R^3} \sum_{i=1}^{\infty} g_{\rm eff}^{2(i-1)} \int_{\pi R^2 T}^{\infty} dr\, r^3\, F^{(i)}\left(\sqrt{\alpha'} RT/\sqrt{l} r\right) ,$$

then a simple rescaling of r leads to

$$f \sim T^4 \,.$$

Note that the horizon resolves the Hagedorn singularity
(if the ‘t Hooft coupling is large enough).