CFT DRIVEN COSMOLOGY AND DGP/CFT CORRESPONDENCE

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4th International Sakharov Conference on Physics, Moscow, 2009

Introduction

CFT (conformal anomaly) driven cosmology

A.A.Starobinsky (1980);
Fischetty,Hartle,Hu;
Riegert; Tseytlin;
Antoniadis, Mazur&Mottola;
……….

New setting of cosmological initial conditions

A.B. & A.Kamenshchik,
JCAP, 09, 014 (2006)
Phys. Rev. D74, 121502 (2006);
A.B., Phys.Rev.Lett.
99, 071301 (2007)

$$|\Psi> \;\Rightarrow\; \hat{\rho}_{\rm microcanonical}$$

$\Lambda_{min} < \Lambda < \Lambda_{max}$ inflation dark energy

cosmological landscape

# of conformal fields (species) N >>1 and 1/N expansion

4D CFT cosmology ⟷ 5D brane induced gravity with Schwarzschild-de Sitter bulk (generalized DGP model)

Plan

CFT driven cosmology – initial conditions via EQG statistical sum:

- constraining landscape of Λ;
- justification from Lorentzian theory --- microcanonical ensemble in cosmology

Cosmological evolution:

- inflation and Big Boost scenario of cosmological acceleration

CFT driven cosmology and the DGP model

DGP/CFT correspondence – background independent duality

AdS vs dS: conformal anomaly uplifting of Λ<0 to Λ>0.

CFT driven cosmology – initial conditions via statistical sum

$$S_E[\,g_{\mu\nu},\phi\,] = -\frac{1}{16\pi G}\int d^4x\, g^{1/2}\left(R-2\Lambda\right) + S_{CFT}[\,g_{\mu\nu},\phi\,]$$

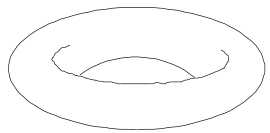
$\Lambda=3H^2$ -- primordial cosmological constant

$N_s \gg 1$ conformal
fields of spin s=0,1,1/2

Statistical sum:

$$e^{-\Gamma} = \int_{\text{periodic}} D[\,g_{\mu\nu},\phi\,]\; e^{-S_E[\,g_{\mu\nu},\phi\,]}$$

on $S^3 \times S^1$



A.B. & A.Yu.Kamenshchik,
JCAP, 09, 014 (2006)
[hep-th/0605132];
Phys. Rev. D74, 121502 (2006)
[hep-th/0611206]

Euclidean FRW metric

3-sphere of a unit size

$$ds^2 = N^2\, d\tau^2 + a^2\, d^2\Omega^{(3)}$$

lapse   scale factor

$$[\, g, \phi\,] = [\, a(\tau), N(\tau); \Phi(x)\,]$$

minisuperspace background

$$\Phi(x) = (\varphi(x), \psi(x), A_\mu(x), h_{\mu\nu}(x), \ldots)$$

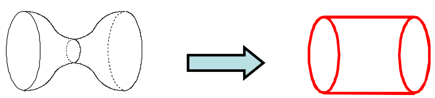
quantum "matter" – cosmological perturbations

$$e^{-\Gamma} = \int_{\text{periodic}} D[\, a, N\,]\, e^{-\Gamma_E[\, a, N\,]}$$

$$e^{-\Gamma_E[\, a, N]} = \int_{\text{periodic}} D\Phi(x)\, e^{-S_E[\, a, N; \Phi(x)\,]}$$

quantum effective action of Φ on minisuperspace background

Assumption of N_{cdf} conformally invariant, N_{cdf} À 1, quantum fields and recovery of the action from the conformal anomaly and the action on a static Einstein Universe



A.A.Starobinsky (1980);
Fischetty,Hartle,Hu;
Riegert; Tseytlin;
Antoniadis, Mazur&Mottola;
……

$$ds^2 = a^2(\eta)(d\eta^2 + d^2\Omega^{(3)}) \Longrightarrow d\bar{s}^2 = d\eta^2 + d^2\Omega^{(3)}$$

conformal time

spin-dependent coefficients

$$g_{\mu\nu}\frac{\delta\Gamma}{\delta g_{\mu\nu}} = \frac{1}{4(4\pi)^2} g^{1/2}\left(\alpha\Box R + \beta E + \gamma C^2_{\mu\nu\alpha\beta}\right)$$

$$\beta = \frac{1}{360}(2N_0 + 11N_{1/2} + 124N_1)$$

Gauss-Bonnet term

Weyl term

N_s # of fields of spin s

$$\Gamma_E = \Gamma_A + \Gamma_{EU}$$

anomaly

static Einstein universe

Exactly solvable model in the leading order of $1/N_{cdf}$ - expansion

Effective Friedmann equation: $\frac{\delta \Gamma_E[a,N]}{\delta N(\tau)} = 0$

amount of radiation constant

$$\frac{1}{a^2} - \frac{a'^2}{a^2} - \frac{B}{2}\left(\frac{1}{a^2} - \frac{a'^2}{a^2}\right)^2 = \frac{\Lambda}{3} + \frac{\mathcal{C}}{a^4}, \qquad \mathcal{C} = \sum_\omega \frac{\omega}{e^{\omega\eta} \pm 1}$$

"bootstrap" equation: $\eta = \eta[a(\tau)]$

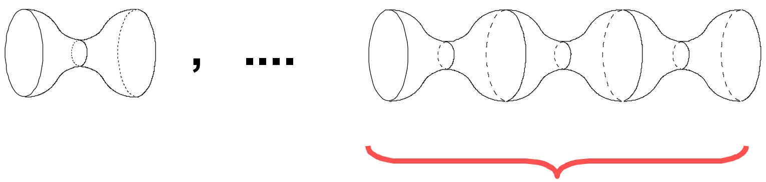
$$B = \frac{3\beta}{4m_P^2}$$

-- coefficient of the Gauss-Bonnet term in the conformal anomaly

$$\eta = \oint d\tau \, \frac{N}{a}$$

instanton period in units of conformal time --- inverse temperature

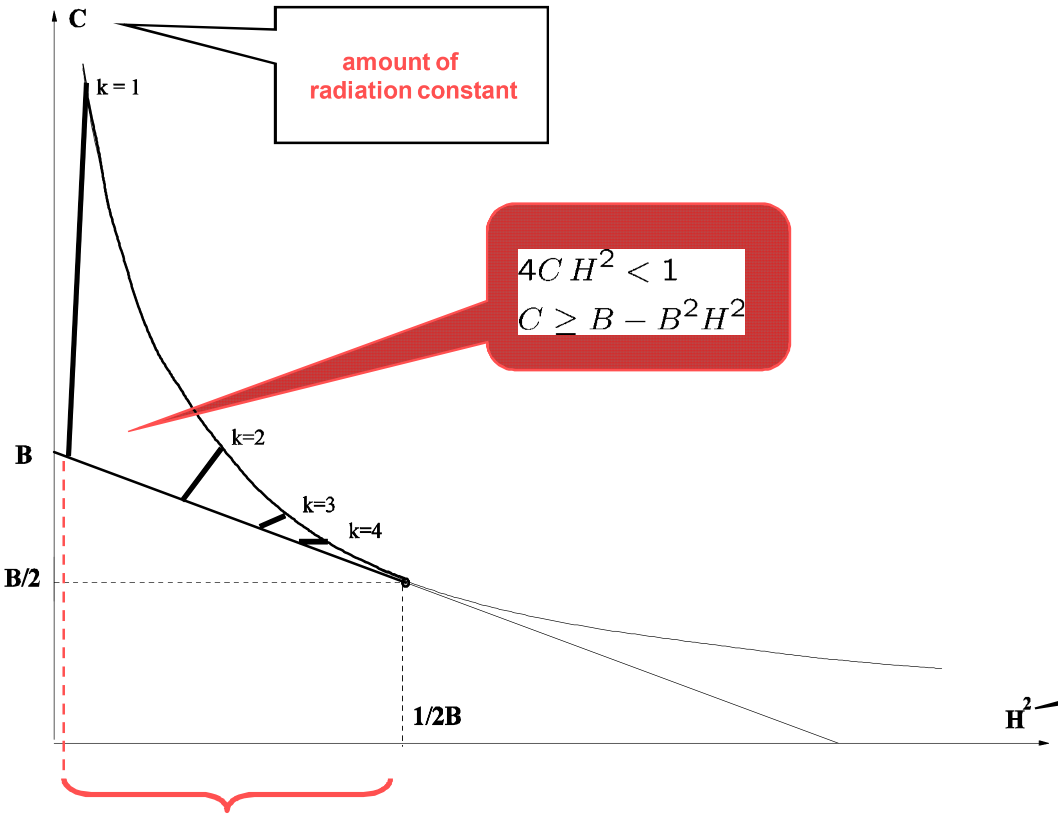
Solutions --- set of tubular periodic garland-type instantons with oscillating scale factor



1- fold, k=1

k- folded garland, k=1,2,3,…

Tree-level version:
Halliwell & Myers (1989);
Fischler, Morgan & Polchinski (1990)



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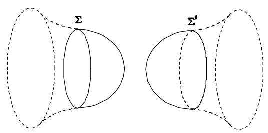
[hep-th/0611206]

- $\Lambda_{\rm min} < \Lambda < \Lambda_{\rm max} = \dfrac{3}{2B}$ **bounded range of the cosmological constant** → **selection rule for string/ cosmologcal landscape**

  ↑ **new QG scale**

- for any $\Lambda > 0$ 

elimination of the no-boundary state from the ensemble due to: $\Gamma = +\infty, \;\; e^{-\Gamma} = 0$

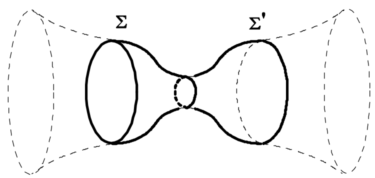
- **# of conformal fields** $\rightarrow N \gg 1; \;\; \Lambda_{\rm min}, \Lambda_{\rm max} \rightarrow \dfrac{\Lambda_{\rm min}}{N}, \dfrac{\Lambda_{\rm max}}{N}$: **1/N-approximation**

Justification from Lorentzian theory --- microcanonical ensemble in cosmology

1. EQG density matrix

$$\rho[\varphi,\varphi'] = e^{\Gamma} \int\limits_{g,\phi|_{\Sigma,\Sigma'}=(\varphi,\varphi')} D[g,\phi]\, e^{-S_E[g,\phi]}$$

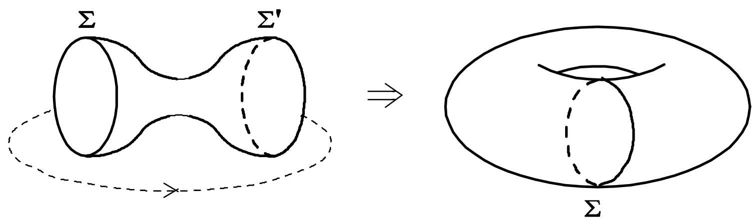
D.Page (1986)



Effective action: statistical sum

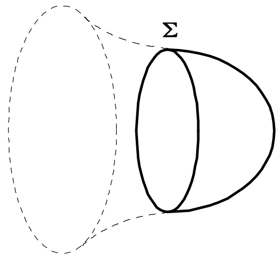
$$e^{-\Gamma} = \int\limits_{g,\phi|_{\Sigma}=g,\phi|_{\Sigma'}} D[g,\phi]\, e^{-S_E[g,\phi]}$$

integration over periodic fields:

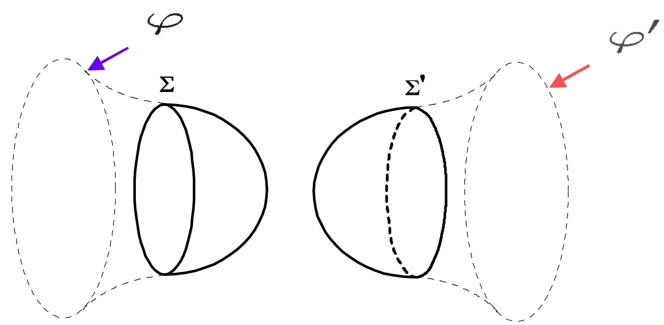


From the pure Hartle-Hawking state to a statistical ensemble – the density matrix:

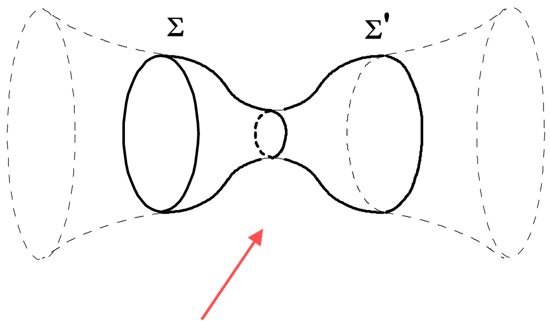
$$|\boldsymbol{\Psi}_{HH}\rangle = \boldsymbol{\Psi}_{HH}[\varphi]$$



$$|\boldsymbol{\Psi}_{HH}\rangle\langle\boldsymbol{\Psi}_{HH}|$$
$$= \boldsymbol{\rho}_{HH}[\varphi, \varphi']$$



$$\hat{\rho}_{\text{mixed}}$$
$$= \boldsymbol{\rho}_{\text{mixed}}[\varphi, \varphi']$$



instanton bridge mediates density matrix correlations

Why Euclidean? Why S^3£ S^1 topology?

2. Microcanonical path integral in cosmology

Canonical (phase-space) path integral in Lorentzian theory:

3-metric and matter fields $q = \big(g_{ij}(\mathbf{x}), \phi(\mathbf{x})\big);\;\; p$ -- conjugated momenta

$$\rho(q_+, q_-) = e^{\Gamma} \int\limits_{q(t_\pm)=q_\pm} D[\,q, p, N\,]\; e^{\,i\int_{t_-}^{t_+} dt\,(p\,\dot{q} - N^\mu H_\mu)}$$

lapse and shift functions (N^μ)

constraints $H_\mu = H_\mu(q, p)$

Range of integration over N^μ: $-\infty < N^\mu < \infty$

Wheeler-DeWitt equations

$$\hat{H}_\mu(q, \partial/i\partial q)\, \rho(q, q_-) = 0$$

Microcanonical density matrix

$$\hat{\rho} \sim \text{“}\Big(\prod_\mu \delta(\hat{H}_\mu)\Big)\text{”}$$

A.O.B., Phys.Rev.Lett. 99, 071301 (2007)

Semiclassical expansion and saddle points:

No periodic solutions of effective equations with real Lorentzian lapse N_L
Saddle points comprise Wick-rotated (Euclidean) geometry:

$$t = \tau, \quad N_L = -iN$$

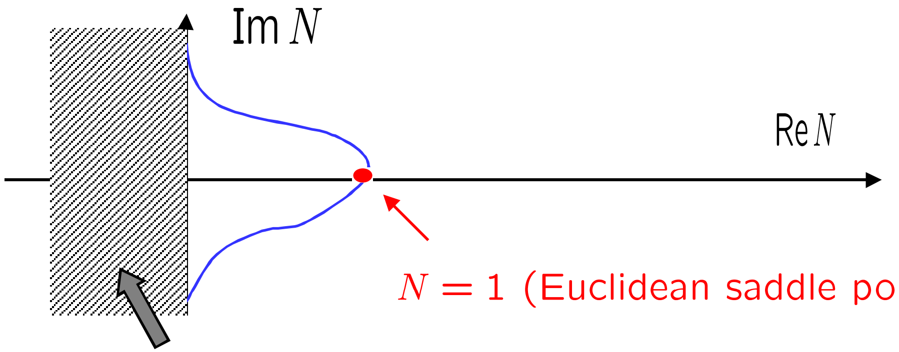
Euclidean lapse

Lorentzian path integral =EQG path integral with the imaginary lapse integration contour:

$$e^{-\Gamma} = \int D[\,a, N\,]\; e^{-\Gamma_E[\,a, N\,]}$$

$$N \in [-i\infty,\, i\infty]$$

conformal "rotation"



$N = 1$ (Euclidean saddle point)

Deformation of the original contour of integration

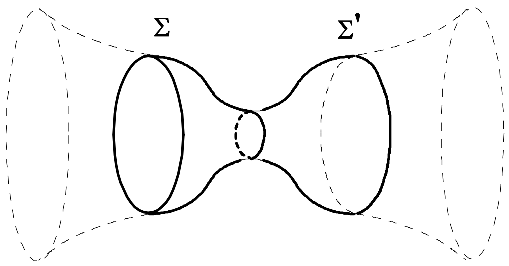
$$-i\infty < N < i\infty$$

into the complex plane to pass through the saddle point

domain of non-analyticity — elimination of $N = -1$ tunneling states

Cosmological evolution from the microcanonical state

Lorentzian Universe with initial conditions set by the saddle-point instanton



Analytic continuation of the instanton solutions:

$$\tau = it, \; a(t) = a_E(it)$$

Decay of a composite Λ in the end of inflation and particle creation of conformally non-invariant matter:

$$\frac{\Lambda}{3} + \frac{\mathcal{C}}{a^4} \Rightarrow \frac{8\pi G}{3}\varepsilon(a)$$

matter energy density

Modified Friedmann equation

$$\frac{\dot{a}^2}{a^2} + \frac{1}{a^2} = \frac{\pi}{\beta G}\left\{1 - \sqrt{1 - \frac{16G^2}{3}\beta\varepsilon}\right\}$$

Coefficient of the Gauss-Bonnet term in conformal anomaly:

$$\beta = \frac{\pi B}{G}$$

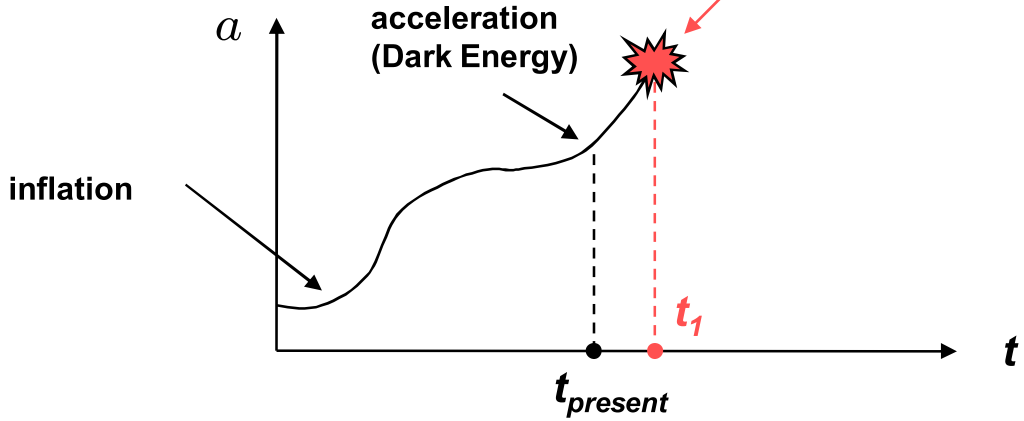
- **Recovery of GR:** $$\frac{\dot{a}^2}{a^2} + \frac{1}{a^2} = \frac{8\pi G}{3}\varepsilon, \quad G^2\varepsilon \ll \frac{1}{\beta}$$

- **Cosmological acceleration and Big Boost singularity**

A.B, C.Deffayet and A.Kamenshchik, JCAP {\bf 05} (2008) 020

Assumption of growing β: $$\ddot{a} \sim \frac{1}{\sqrt{1 - 16G^2\beta\varepsilon/3}} \to +\infty$$

Big Boost singularity: $$\ddot{a}_\infty = \infty, \quad H_\infty^2 = \frac{\pi}{\beta G} < \infty$$



CFT cosmology vs DGP model

Generalized cosmological DGP model with Λ_5 , bulk black hole of the mass » C and matter vacuum on the brane

Euclidean action

$$S_{DGP}[G_{AB}(X)] = -\frac{1}{16\pi G_5}\int_{\rm Bulk} d^5X\, G^{1/2}\left(R^{(5)}(G_{AB}) - 2\Lambda_5\right)$$
$$-\int_{\rm brane} d^4x\, g^{1/2}\left(\frac{1}{8\pi G_5}[K] + \frac{1}{16\pi G_4}R(g_{\mu\nu})\right).$$

5D Schwarzschild-dS solution with a bulk black hole of the mass » R_s^2/G_5

$$ds^2_{(5)} = f(R)dT^2 + \frac{dR^2}{f(R)} + R^2 d\Omega^2_{(3)}$$
$$f(R) = 1 - \frac{\Lambda_5}{6}R^2 - \frac{R_S^2}{R^2}$$

embedding

$$ds^2_{(4)} = d\tau^2 + a^2(\tau)d\Omega^2_{(3)}$$

$$R = a(\tau)$$
$$T = T(\tau)$$
$$T'(\tau) = \frac{\sqrt{f(a) - a'^2}}{f(a)}$$

Brane dynamics equation from Israel junction condition:

$$\left(K_{\mu\nu} - K g_{\mu\nu}\right)_{\mathbf{b}} = -r_c \left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R\right), \quad r_c = \frac{G_5}{2\,G_4}$$

DGP crossover scale
etween 4D and 5D phases

5D DGP side

$$r_c^2 \left(\frac{1}{a^2} - \frac{a'^2}{a^2}\right)^2 = \frac{1}{a^2} - \frac{a'^2}{a^2} - \frac{\Lambda_5}{6} - \frac{R_S^2}{a^4}$$

$$\begin{cases} B = 2r_c^2, \quad \Lambda_4 = \dfrac{\Lambda_5}{2} \\ \mathcal{C} = R_S^2 \end{cases}$$

4D CFT cosmology

$$\frac{B}{2} \left(\frac{1}{a^2} - \frac{a'^2}{a^2}\right)^2 = \frac{1}{a^2} - \frac{a'^2}{a^2} - \frac{\Lambda_4}{3} - \frac{\mathcal{C}}{a^4}$$

C **» mass of the 5D black hole**

Dynamical equations coincide, but is there a bootstrap equation for *C* ?
Yes, there is --- from the absence of conical singularities in the bulk.

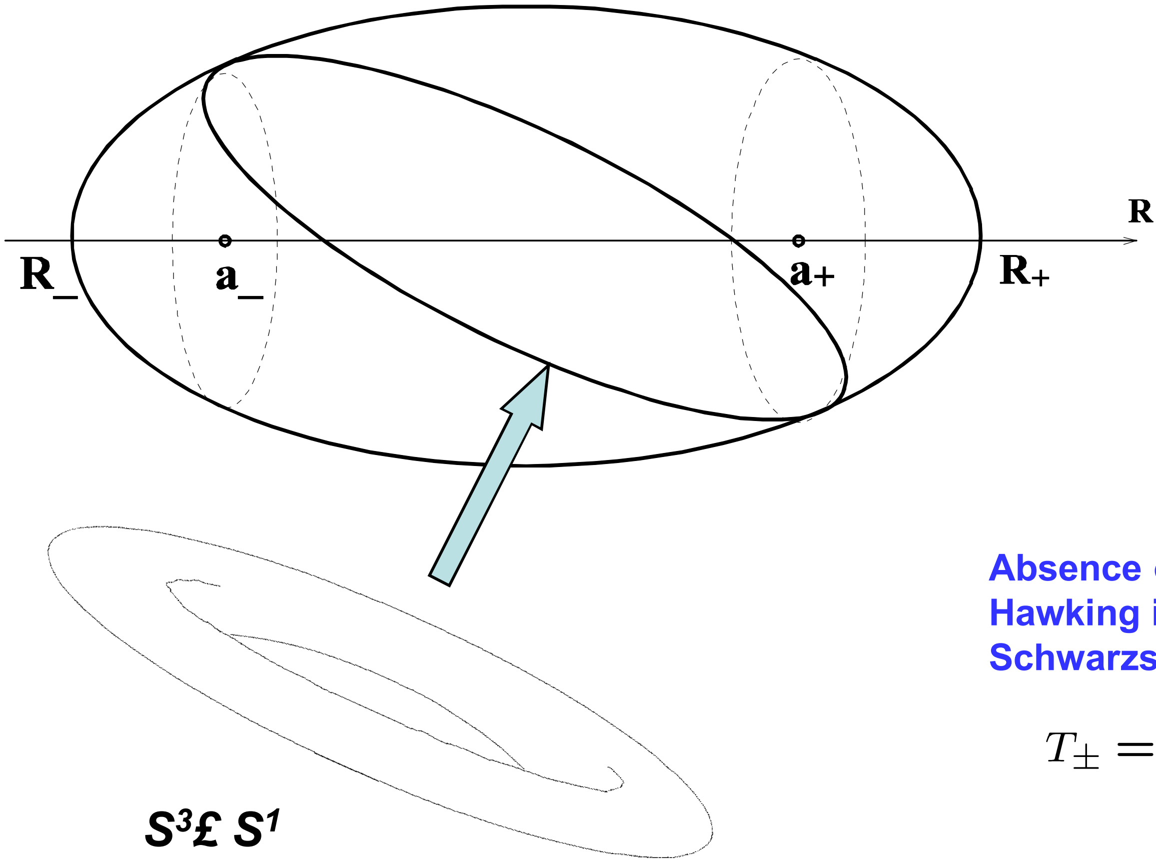
Euclidean Schwarzschild-dS “cigar” instanton:

$$f(R) \geq 0, \quad R_- \leq R \leq R_+$$

$$R_\pm^2 = \frac{3}{\Lambda_5}\left(1 \pm \sqrt{1 - 2\Lambda_5 R_S^2/3}\right)$$

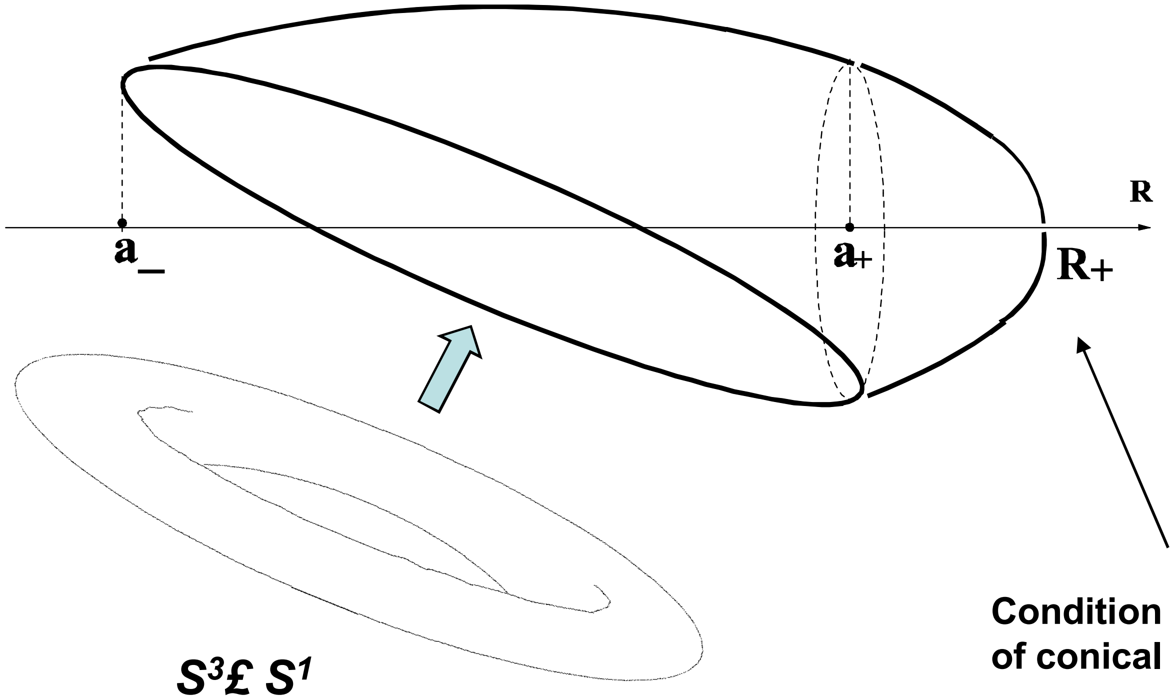
$$R_- < \underbrace{a_- \leq a(\tau) \leq a_+}_{\text{4D instanton domain}} < R_+$$

4D instanton domain



Absence of conical singularities at R§ :
Hawking inverse temperatures of
Schwarzschild and dS horizons:

$$T_\pm = \frac{4\pi}{f'(R_\pm)}, \quad T_+ \neq T_-$$



Condition of absence of conical singularity:

$$\oint d\tau\, T'(\tau) \equiv 2k \int_{a_-}^{a_+} da \frac{\sqrt{f(a) - a'^2}}{a' f(a)} = \frac{4\pi}{|df(R_+)/dR_+|}$$

This gives the equation alternative to the CFT bootstrap:

$$\Updownarrow$$

$$\mathcal{C} = \sum_\omega \frac{\omega}{e^{\omega\eta} \pm 1}$$

For selected values of Λ both bootstrap equations yield the same instantons ---
complete duality of 4D and 5D pictures.

Numerical analysis $\Longrightarrow$

$$\frac{2}{3}B\Lambda = \begin{cases} 0.36, & s = 0 \\ 0.986, & s = 1/2 \\ 0.9998, & s = 1 \end{cases}$$

CFT side: saturation of the new QG scale limit ---- effective theory cutoff in the model with N>>1 species

$$\Lambda_{\rm max} = \frac{3}{2B} \sim \frac{m_P^2}{N}$$

G.Dvali,
hep-th:0706.2050

5D side: limit of a large semiclassical BH

$$R_S^2 \sim \frac{N}{m_P^2} \gg \frac{1}{m_P^2},$$

DGP/CFT correspondence: towards background independent duality

Does this duality extend beyond cosmological model?

Israel junction condition in DGP model →

$$\begin{cases} K_{\mu\nu} = -r_c\left(R_{\mu\nu} - \dfrac{1}{6} g_{\mu\nu} R\right), \\ K_{\mu\nu}^2 - K^2 = r_c^2\left(R_{\mu\nu}^2 - \dfrac{1}{3} R^2\right) \end{cases}$$

Constraint equation in the bulk:

$$R^{(4)} + K_{\mu\nu}^2 - K^2 - 2\Lambda_5 = 0$$

⇓

$$R^{(4)} + r_c^2\left(R_{\mu\nu}^2 - \frac{1}{3} R^2\right) - 2\Lambda_5 = 0$$

DGP side

Trace equation $g_{\mu\nu}\frac{\delta\Gamma}{\delta g_{\mu\nu}} = 0$

$$E = C^2_{\mu\nu\alpha\beta} - 2\left(R^2_{\mu\nu} - \frac{1}{3}R^2\right)$$

CFT side

$$R^{(4)} + \frac{\beta G_4}{2\pi}\left(R^2_{\mu\nu} - \frac{1}{3}R^2\right) - 4\Lambda_4 - \underbrace{\frac{\beta+\gamma}{4\pi}}_{=\,0} G_4\, C^2_{\mu\nu\alpha\beta} = 0$$

vanishes on FRW

SUSY

$D = 4\ SU(\mathbb{N})\ \mathcal{N} = 4$ SYM

$(N_0, N_{1/2}, N_1) = (6\mathbb{N}^2, 4\mathbb{N}^2, \mathbb{N}^2)$

$B = 3\mathbb{N}^2/8m_P^2$

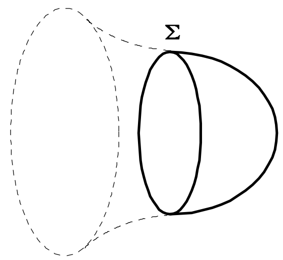
DGP side

$$R^{(4)} + r_c^2\left(R^2_{\mu\nu} - \frac{1}{3}R^2\right) - 2\Lambda_5 = 0$$

AdS vs dS: conformal anomaly uplifting of Λ<0 to Λ>0

CFT cosmology with a negative Λ_4

No-boundary case



$$\Lambda = -3H^2 < 0$$

$$a(\tau) = \frac{1}{H_{\rm eff}} \sin(H_{\rm eff}\tau)$$

$$H_{\rm eff}^2 = \frac{2H^2}{\sqrt{1+2BH^2}-1} > 0$$

$$\Lambda_{\rm eff} = -\frac{2}{3}\frac{\Lambda}{\sqrt{1-2B\Lambda/3}-1} > 0$$

Genuine effect of quantum conformal anomaly!

No KKLT uplifting is needed

Unfortunately:

$$\Gamma = +\infty$$

$$e^{-\Gamma} = 0$$

Tunneling alternative?
Linde; Rubakov; Zeldovich&Starobinsky; Vilenkin (1984)

$$e^{+\Gamma} = \infty$$

Conclusions

Microcanonical state in the CFT driven cosmology with a large # of quantum fields N:

Initial conditions for inflation with a limited range of Λ --- cosmological landscape and Big Boost mechanism of DE at late stages of expansion

Dual 5D description via the DGP model with $\Lambda_5>0$ and 5D BH imitating radiation on the brane --- for $N>>1$ semiclassical BH vs stronggly coupled CFT at the cutoff scale $m_P/N^{1/2}$

Indication of a background-independent duality for superconformal models

Conformal anomaly uplifting to $\Lambda_{eff}>0$ reconciling a negative primordial Λ with inflation