On Higher Spin Interactions with a Scalar Matter Field

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based on X. B. , E. Joung and J. Mourad, arXiv:0903.3338 [hep-th].

Perturbative & Power counting: UV behaviour

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But *not* always for an *infinite* set of fields with unbounded spin.

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String theory properties:

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Conclusion:
Adding an infinite number of problems with increasing difficulty can be a solution!

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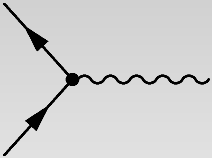
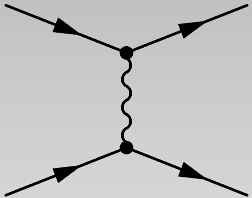
Strategy

Make use of the known propagators and cubic vertices including:

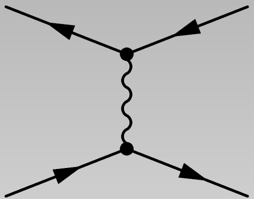
- scalar matter field (straight lines) and
- higher-spin gauge field (curly line).



Compute the tree-level exchange amplitude when the interaction is mediated by a massless higher-spin particle in the elastic scattering process $\phi\,\phi\;\rightarrow\;\phi\,\phi$



or in the elastic scattering process $\phi\,\bar{\phi} \rightarrow \phi\,\bar{\phi}$



Plan of the talk

Klein-Gordon action

$$\mathcal{S}_0^{\mathsf{kin}}[\,\phi\,] \;=\; -\frac{1}{2}\int d^n x\,\left(\,\eta^{\mu\nu}\,\partial_\mu\phi^*(x)\,\partial_\nu\phi(x) + m^2\,\phi^*(x)\,\phi(x)\,\right)\,,$$

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$\Rightarrow$ *Scalar field propagator* $= \;\frac{1}{p^2+m^2}\;.$

Quadratic action

$$\mathcal{S}_2^{\mathsf{kin}}[\,h\,] = -\sum_{S\geqslant 0}\frac{1}{2\,S!}\int d^n x\,\overset{(S)}{h}{}_{\mu_1\ldots\mu_S}(x)\,\Box\,\overset{(S)}{h}{}^{\mu_1\ldots\mu_S}(x)\;+\;\ldots$$

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<u>Unconstrained formalism</u> (Francia, Mourad, Sagnotti; 2007)
No trace constraints $\Rightarrow$ easier to couple with currents

Minimal coupling

$$\mathcal{S}_1^{\text{min}}[\,\phi, h\,] = -\sum_{S\geqslant 0} \frac{c_S}{S!} \int d^n x \; \overset{(S)}{h}{}_{\mu_1\ldots\mu_S}(x)\, J^{\mu_1\ldots\mu_S}(x)$$

Arbitrary coupling constants $c_S \in \mathbb{R}$

Gauge invariance of the action

$$\mathcal{S}[\,\phi, h\,] \,=\, \mathcal{S}_0^{\text{kin}}[\,\phi\,] \,+\, \mathcal{S}_1^{\text{min}}[\,\phi, h\,] \,+\, \mathcal{S}_2^{\text{kin}}[\,h\,] \,+\, \text{higher}\,.$$

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$$\Rightarrow \quad \partial_{\mu_1} J^{\mu_1\ldots\mu_S}(x) \;\propto\; \text{Klein-Gordon equation}$$

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Minimal coupling of gauge fields with
conserved currents for the matter field

Conserved current

Set of symmetric conserved currents of all ranks
(Berends, Burgers, van Dam; 1986)

$$J_{\mu_1 \dots \mu_S}(x) = \left(\frac{i}{2}\right)^S \phi(x) \overleftrightarrow{\partial}_{\mu_1} \cdots \overleftrightarrow{\partial}_{\mu_S} \phi^*(x)$$

- Real
- Bilinear in the complex scalar field
- Number of derivatives = Rank

Cubic vertex

$$\begin{aligned} S_1[\,\phi, h^{(S)}\,] &= \frac{c_S}{S!}\int d^n x \; \overset{(S)}{h}{}_{\mu_1\ldots\mu_S}(x)\, J^{\mu_1\ldots\mu_S}(x) \\ &= -\int \frac{d^n\ell}{(2\pi)^n}\,\frac{d^n k}{(2\pi)^n}\,\phi^*(\ell)\;\phi(k)\;\overset{(S)}{h}{}_{\mu_1\ldots\mu_S}(\ell - k)\,\times \\ &\qquad \times\,\frac{c_S}{S!}\left(\frac{k^{\mu_1}+\ell^{\mu_1}}{2}\right)\ldots\left(\frac{k^{\mu_S}+\ell^{\mu_S}}{2}\right). \end{aligned}$$

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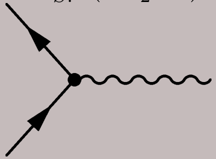
$\Rightarrow$ *Cubic vertex* $= \;-\,\frac{c_S}{S!}\,\left(\frac{k^{\mu_1}+\ell^{\mu_1}}{2}\right)\ldots\left(\frac{k^{\mu_r}+\ell^{\mu_S}}{2}\right).$



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- θ = Scattering angle, determined by

$$\sin^2(\theta/2) = -t/(s - 4\,m^2)\,, \quad \cos^2(\theta/2) = -u/(s - 4\,m^2)$$

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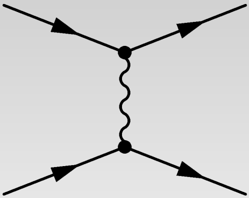
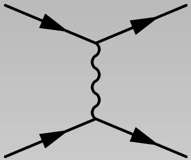
$$\overset{(S)}{\mathcal{A}}(s,t,u) = \underline{t\text{-channel spin-}S\text{ exchange amplitude}}$$

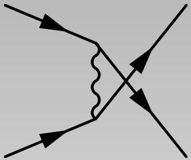


For bosons, the total amplitude for the scattering process $\phi(k_1)\,\phi(k_2) \,\rightarrow\, \phi(\ell_1)\,\phi(\ell_2)$ contains the sum of the t and u channel amplitudes:

$$\overset{(S)}{\mathcal{A}}_{\text{total}}(\phi\,\phi \,\rightarrow\, \phi\,\phi) \quad = $$



$$+$$



$$= \quad \overset{(S)}{\mathcal{A}}(s,t,u) \quad + \quad \overset{(S)}{\mathcal{A}}(s,u,t)\,.$$

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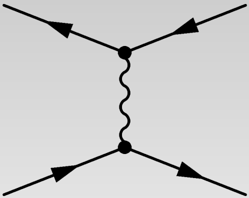
- In higher dimensions ($n \geqslant 6$), the amplitude can be expressed in terms of Gegenbauer polynomials $C_S^{\frac{n}{2}-2}$.

Crossing

Elastic scattering $\phi(k_1)\,\bar{\phi}(k_2) \rightarrow \phi(\ell_1)\,\bar{\phi}(\ell_2)$

$$\overset{(S)}{\mathcal{A}}(u,t,s) = (-1)^S\,\overset{(S)}{\mathcal{A}}(s,t,u)$$



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- Regge limit (s large, t fixed)

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- Fixed scattering-angle limit (s and t large, t/s fixed)

$$\overset{(S)}{\mathcal{A}}(s,t,u) \sim -\,\frac{1}{4}\,\frac{a_S}{S!}\,\Big(-\frac{\ell_P^2}{8}\,\sin^2(\theta/2)\,s\Big)^{S-1}\,T_S\left(\frac{1+\cos^2(\theta/2)}{\sin^2(\theta/2)}\right)\,.$$

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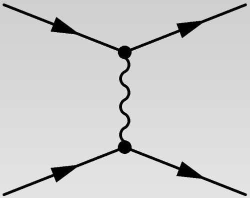

Summation of the amplitudes for all spins

The total amplitude including all possible gauge boson exchanges is the (possibly infinite) sum

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Let us denote by $a(z)$ the generating function of the coefficients $a_S \geqslant 0$, in the sense that

$$a(z) := \sum_{S\geqslant 0} \frac{a_S}{S!}\; z^S \,.$$

Summation of the amplitudes for all spins

Exact sum

$$\mathcal{A}(s,t,u) = -\frac{1}{\ell_P^2\, t}\left[a\Big(-\frac{\ell_P^2}{8}\big(\sqrt{s}+\sqrt{-u}\,\big)^2\Big)+\right.$$
$$\left.+a\Big(-\frac{\ell_P^2}{8}\big(\sqrt{s}-\sqrt{-u}\,\big)^2\Big)-a_0\right].$$

Summation of the amplitudes for all spins

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Remark: $a(z)$ analytic around the origin $\Longrightarrow$ $\mathcal{A}(s,t,u)$ also is

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High-energy regime $s \gg \ell_P^{-2} \gg m^2$

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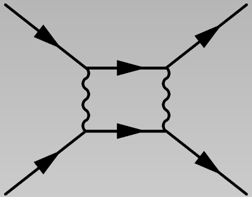
Example: Box diagram

Box diagram



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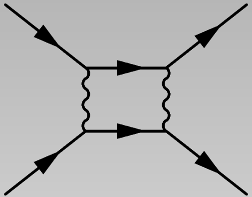

is proportional to

$$\int d^4p \, \frac{\mathcal{A}\Big(\phi(k_1)\phi(k_2) \to \phi(k_1+p)\phi(k_2-p)\Big)\, \mathcal{A}\Big(\phi(k_1+p)\phi(k_2-p) \to \phi(\ell_1)\phi(}{\Big((k_1+p)^2+m^2\Big)\Big((k_2-p)^2+m^2\Big)}$$

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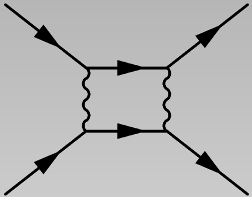

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and is UV finite if $a(z)$ goes to some constant when $z \to \pm\infty$.

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Of course, this does not imply that the corresponding total one-loop amplitudes are finite because other diagrams should be taken into account, some of which might include higher-order vertices which are not considered in the present paper.
Nevertheless, it is already suggestive to observe that some Feynman diagrams may be UV finite if all contributions of the whole infinite tower of gauge fields are summed and if the coupling constants c_S behave nicely for large spin S .

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    - Feynman $\longrightarrow$ Witten diagrams
    - Test AdS_4/CFT_3 higher-spin/O(N)-model conjecture (Klebanov & Polyakov 2002, Petkou 2003, Sezgin & Sundell 2003, Leonhardt-Manvelyan-Rühl 2004, ...)