Heat and Gravbitation

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The problem: The cooling of stars

"They do NOT become Black Holes"

The main idea: Replace Tolman's formula:

$$\mathbf{G}_{\mu\nu}/8\pi\mathbf{G} = \mathbf{T}_{\mu\nu} = \rho\,\mathbf{U}_\mu\mathbf{U}_\nu - \mathbf{p}\ \mathbf{g}_{\mu\nu}, \qquad \mathbf{g}^{\mu\nu}\mathbf{U}_\mu\mathbf{U}_\nu = \mathbf{1}$$

by an action principle.

WHY?

1. Objections to Tolman's formula::

a. The normalization condition is appropriate for free particles. It is an "on shell" constraint.

b. Not enough variables. There is no provision dealing with the photon gas, or mixtures.

c. The equation of continuity, a corner stone of the non-relativistic theory, is not respected.a This implies confused boundary conditions, np concept of chemical potential.

d. Interpretation of "mass density" is hazardous.

Other reasons to favor an action principle:

2. Recall the use of action principles in other fields.

Importance of "off shell" formulation.

Internal consistency, including Onsager relations.

3. Operational definition of energy.

Hamiltonian functional needed to prove stability.

Replacing Tolman's formula:

$$\mathbf{G}_{\mu\nu}/\mathbf{8}\pi\mathbf{G} = \mathbf{T}_{\mu\nu} = \rho\,\mathbf{U}_\mu\mathbf{U}_\nu - \mathbf{p}\ \mathbf{g}_{\mu\nu},$$

by an action principle is easy. Simplest example:

$$\mathbf{A} = \mathbf{R}(\mathbf{g})/\mathbf{8}\pi\mathbf{G} + \int \mathbf{d}^4\mathbf{x}\ (\ \frac{\rho}{\mathbf{2}}(\mathbf{g}^{\mu\nu}\psi_{,\mu}\psi_{,\nu} - \mathbf{c}^2) - \mathbf{V}(\rho, \mathbf{T})\ ]$$

Features: Conserved current,

Familiar non-relativistic approximation. The non-relativistic velocity potential Φ appears in the expansion

$$\psi = c^2 t + \Phi.$$

Sources of wisdom that have been consulted so far.

Astrophysics.

Thermodynamics, equilibrium only.

Hydrodynamics and radiation hydrodynamics.

Thermodynamics of non stationary processes.

Material science.

With all that, I still do not have much information about liqiods.

The ideal gas

Thermodynamics was built on it.

Let us follow the example.

Adiabatic lagrangian for an ideal gas

$$\mathbf{L}[\mathbf{\Phi}, \rho, \mathbf{T}] = \int \mathbf{d^3x}\mathcal{L},$$

$$\mathcal{L} = \rho(\dot{\Phi} - \vec{v}^{\,2}/2 - \phi + \lambda) - \mathcal{R}T\rho \log \frac{\rho}{T^n k_0},$$

$\rho =$ **density,**

$\Phi =$ **velocity potential,** $\vec{v} = -$**grad** Φ**,**

$T =$ **temperature,**

$n =$ **adiabatic index,**

$k_0 =$ **constant (entropy),**

$\lambda =$**Lagrange multiplier (chemical potential),**

$\phi =$ **gravitational potential.**

Lagrangian for an ideal gas:

$$\mathcal{L} = \rho(\dot{\mathbf{\Phi}} - \vec{\mathbf{v}}^{\,\mathbf{2}}/\mathbf{2} - \phi + \lambda) - \mathcal{R}\mathbf{T}\rho \log \frac{\rho}{\mathbf{T^n k_0}} + \mathbf{sT},$$

This simple expression contains the following information.

1. **The equation of continuity (variation of $\mathbf{\Phi}$),**
2. **Bernoulli's equation of motion (variation of ρ),**
3. **The gas law: $\mathbf{p} = \mathcal{R}\mathbf{T}\rho$ (variation of ρ)**
4. **The polytropic relations (variation of T),**
5. **The constant lapse rate of atmospheres,**
6. **The formula for the internal energy of an ideal gas,**
7. **The entropy, the free energy and the Gibbs free energy.**

Some details.

Continuity: $\dot{\rho} + \text{div}(\vec{v}\rho) = 0$

Bernoulli: $\rho \frac{\mathbf{D}}{\mathbf{Dt}} \vec{\mathbf{v}} + \text{grad}\ \phi = -\text{grad}\ \mathbf{p}, \quad p = \mathcal{R} T \rho$

Polytropic: $\log \frac{\rho}{T^n k_0} = n$

Lapse rate: $\lambda - \phi = \mathcal{R} T (n+1)$

Internal energy density (on shell): $u = n \mathcal{R} T \rho$

Project: "Lagrangian for H_2O".

"Once it is known there should be no further appeal to kinetic theory"

Entropy.

The lagrangian potential may be interpreted as the free energy density, for the isolated system.

$$\mathcal{L} = \rho(\dot{\mathbf{\Phi}} - \vec{\mathbf{v}}^{\,2}/\mathbf{2} - \phi + \lambda) - \mathcal{R}\mathbf{T}\log\frac{\rho}{\mathbf{T^n k_0}} + \mathbf{sT},$$

The parameter k_0 determines the internal entropy density and sT represents external influence capable of changing the value of k_0.

Lagrangian for ideal gas:

$$\mathcal{L} = \rho(\dot{\mathbf{\Phi}} - \vec{\mathbf{v}}^{\,2}/\mathbf{2} - \phi + \lambda) - \mathcal{R}\mathbf{T}\log\frac{\rho}{\mathbf{T^n k_0}} - \frac{\mathbf{a}}{\mathbf{3}}\mathbf{T^4} + \mathbf{ST},$$

Stefan-Boltzmann term added.

Application 1.

The Stefan-Boltzmann term modifies the effective adiabatic index at very high temperatures, moving it asymptotically towards n = 3. (Compare Eddington, Chandrasekhar.)

$u = n\mathcal{R}T\rho + aT^4, \quad p = \mathcal{R}T\rho + \frac{a}{3}T^4$

Again, the Lagrangian for an ideal gas,

$$\mathcal{L} = \rho(\dot{\mathbf{\Phi}} - \vec{\mathbf{v}}^{\,\mathbf{2}}/\mathbf{2} - \phi + \lambda) - \mathcal{R}\mathbf{T}\rho \log \frac{\rho}{\mathbf{T^n k_0}} - \frac{\mathbf{a}}{\mathbf{3}}\mathbf{T^4} + \mathbf{Ss},$$

Application 2.

Role of radiation in creating the adiabatic atmosphere?

Answer: Prevents cooling by holding k_0 fixed.

See Figure

The isolated atmosphere is NOT isothermal, but adiabatic. Apologies to Maxwell and Boltzmann.

Two proposed experiments,

1. Measure lapse rate in centrifuge.

2. Test concept of universal lapse rate in earth's atmosphere.

Application 3.

Mixing, by adding lagrangians, including mixed-gas atmospheres. This is done by employing the Gibbs function, after adding a surface term to the lagrangian. In the relativistic context this term is the cosmological constant, correctly interpreted as pressure; it is not energy!

Disassociation (likewise)

Phase changes (under study)

There is little that is radically new in these applications, except that the interface with General Relativity is natural (though not yet completely explored).

All have important applications to stellar structure.

Application 4.

Stability: conservation law is important for boundary conditions.

Exact virial theorem:

Particle virial theorem: $\vec{r} \cdot \vec{p}$**.**

Continuum virial theorem: $\rho\Phi$

The energy is the hamiltonian, a well defined functional of the dynamical variables. This new virial theorem can be used to prove the well known fact that $n = 3$ **is the critical value, for stability, of the polytropic index.**

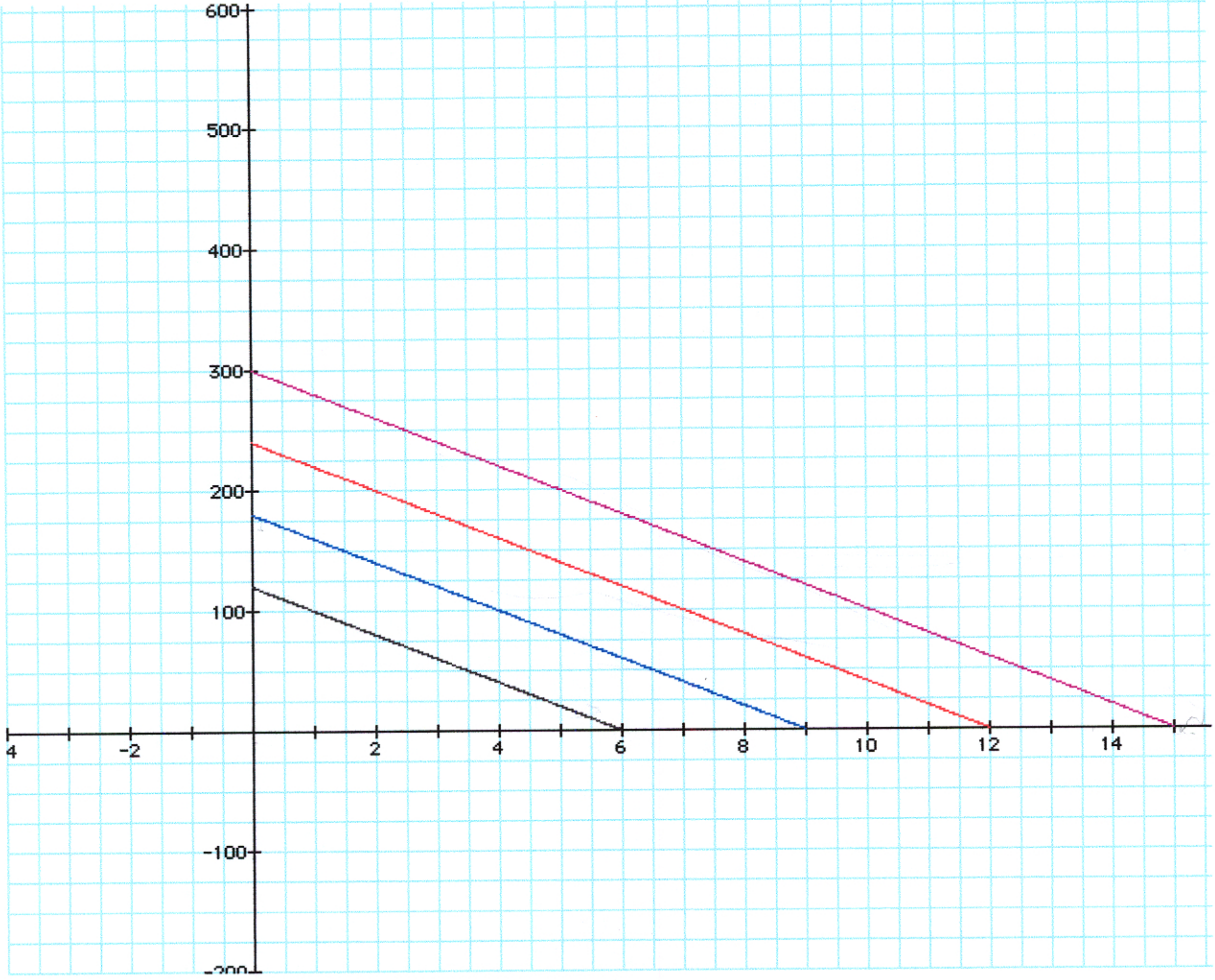
Summary

I have challenged the view that the basics of astrophysics were definitively settled long ago.

Coming from an outsider this will be met with skepticism.

I ask you to communicate your criticism to me. I have come here in the hope that you will do so.

Thank you.



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