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Fractional Charges on Frustrated Lattices

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Known examples of excitations with fractional charges:

(1) in one dimension

⟹ trans-polyacetylene

(2) in two dimensions

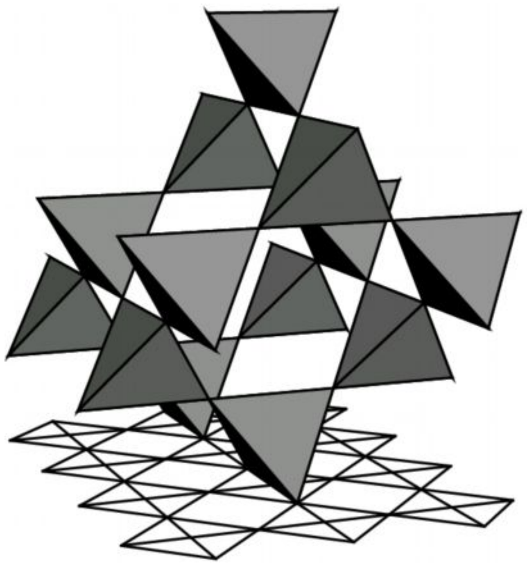
⟹ fractional quantum Hall effect

here: fractional charges in three dimensions

⟹ geometrically frustrated lattices

question:	are there deconfined fractional charges in a three dimensional system?
related question:	do fractional charges always imply fractional statistics?
answer:	a simple Hamiltonian is provided with deconfined fractional charges on a pyrochlore lattice

consider a pyrochlore lattice at half-filling with fully spin-polarized electrons (spinless fermions)



tetrahedron rule
2 empty + 2 occupied sites

pyrochlore → checkerboard

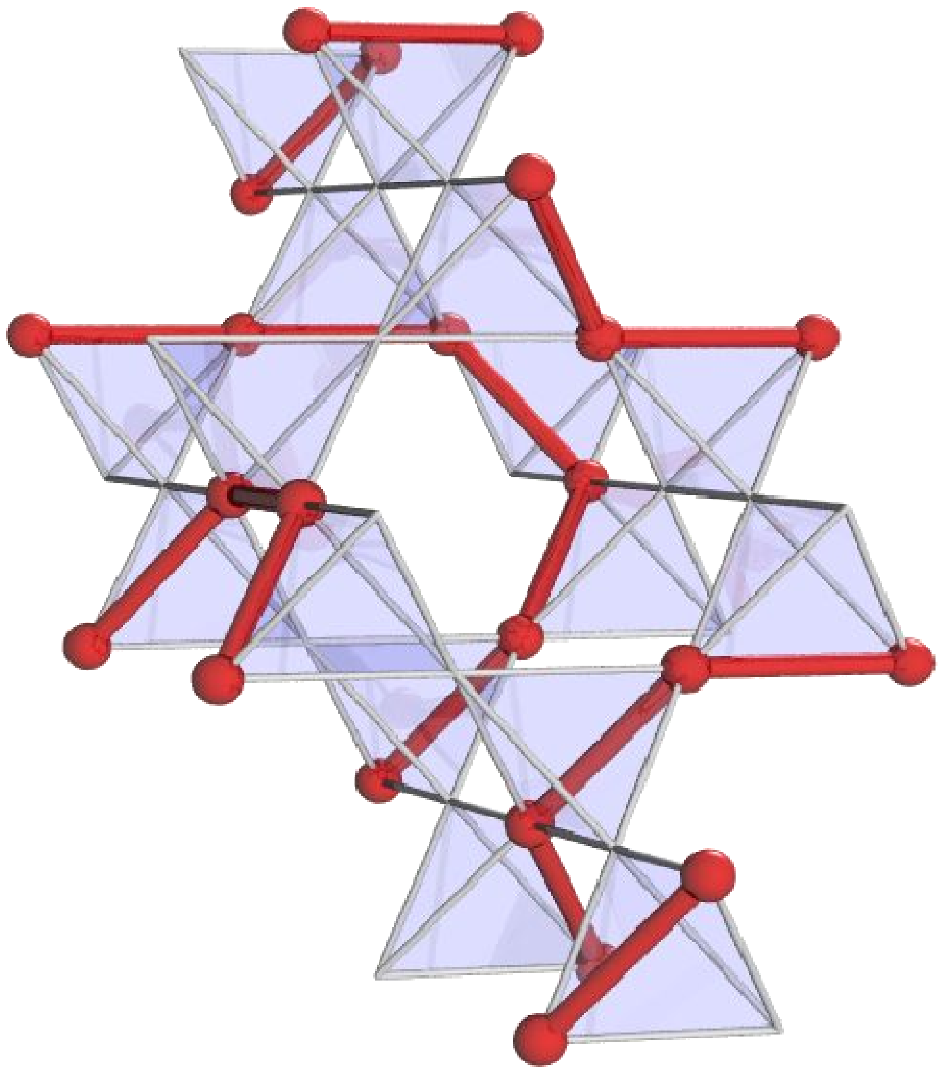
$$H = -\, t \sum_{\langle ij \rangle} \left(c_i^{+} c_j + \text{h.c.} \right) + V \sum_{\langle ij \rangle} n_i n_j$$

strong coupling limit   V ?   t

motivation:

strong electron correlations in spinels AB_2O_4:   e.g., LiV_2O_4, Fe_3O_4

strong non-local constraint!



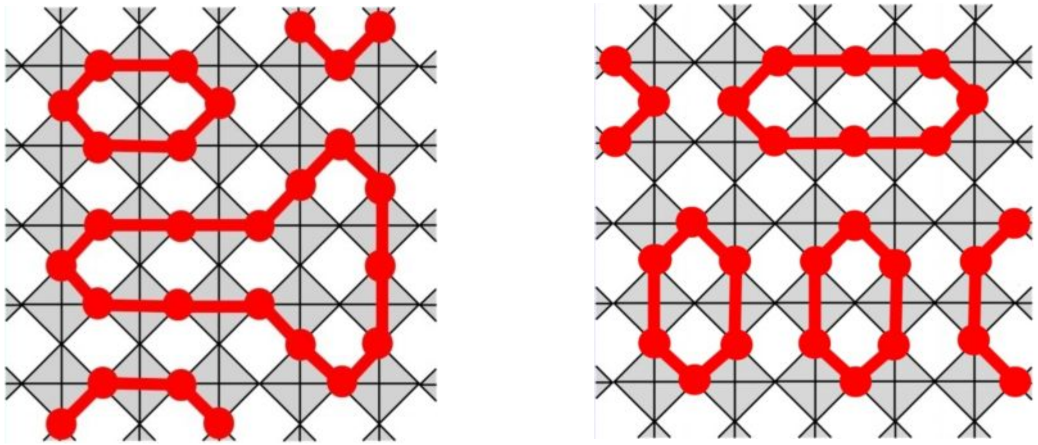
Checkerboard lattice: $t = 0$

ground-state degeneracy: $N_{deg} = (4/3)^{\frac{3}{4}N}$ N = number of sites

two configurations (examples): → loops

related to ice model

(Pauling)



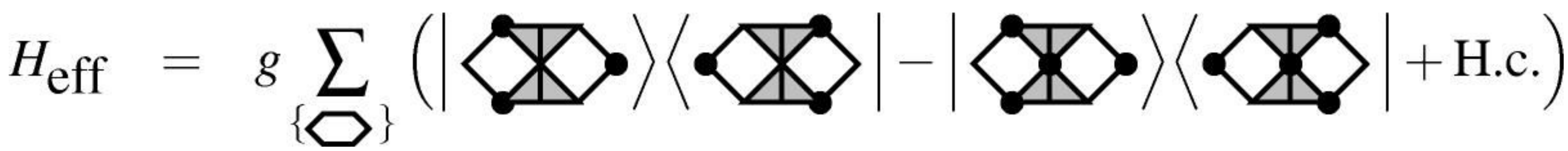
solid lines connect occupied sites

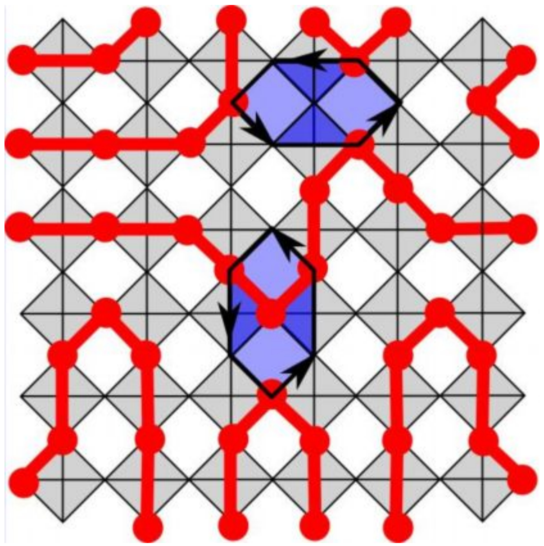
when dynamics is added → simple version of a string theory !

finite hopping $t \neq 0$:

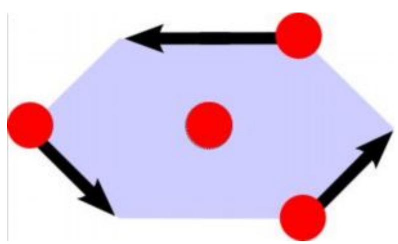
2nd order: constant energy contribution

3rd order: effective Hamiltonian $g = \frac{12t^3}{V^2}$ (sign is irrelevant)

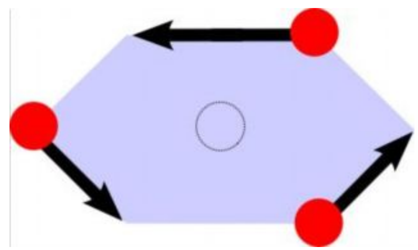




relative sign problem:



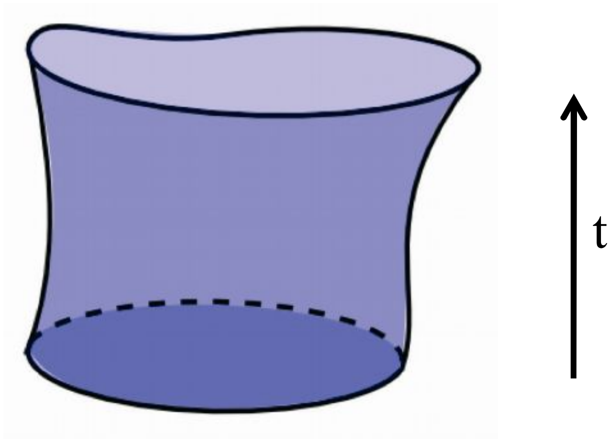
$|B\rangle\langle\bar{B}|$



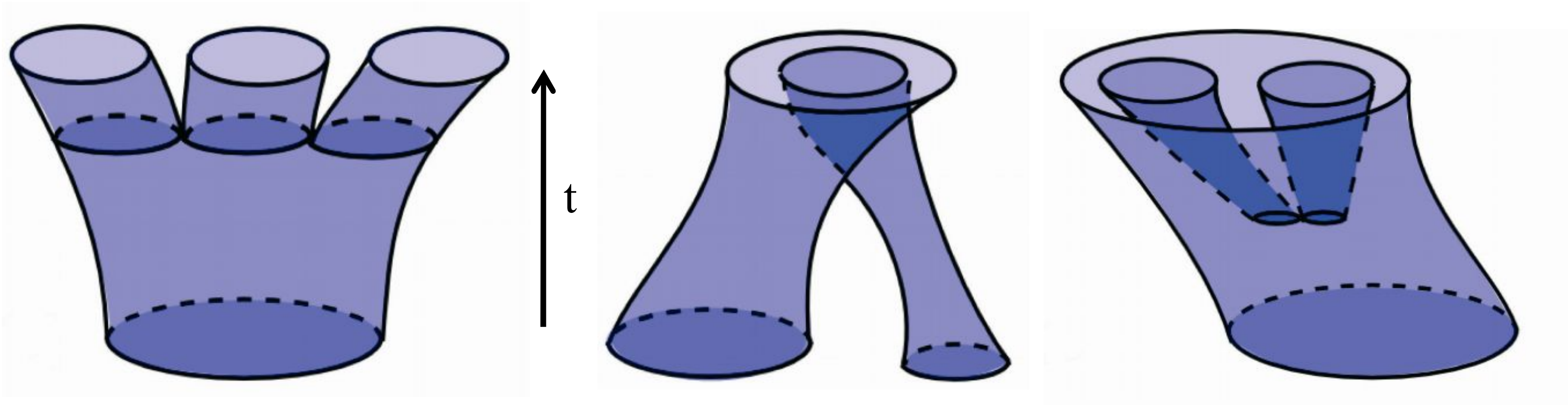
$|A\rangle\langle\bar{A}|$

lifting of the macroscopic degeneracy

Continuum representation of loop dynamics due to H_{eff}

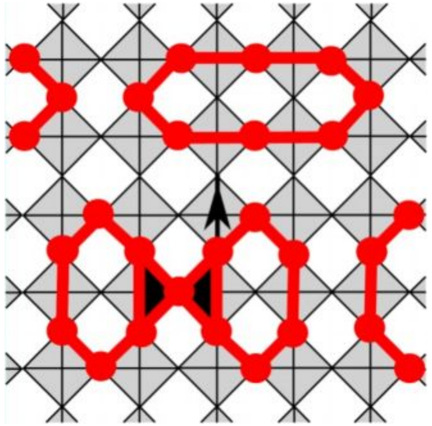


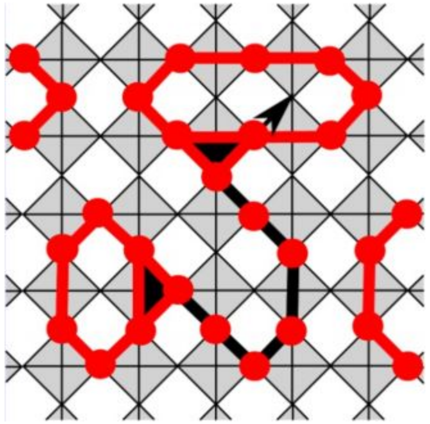
time evolution due to B processes



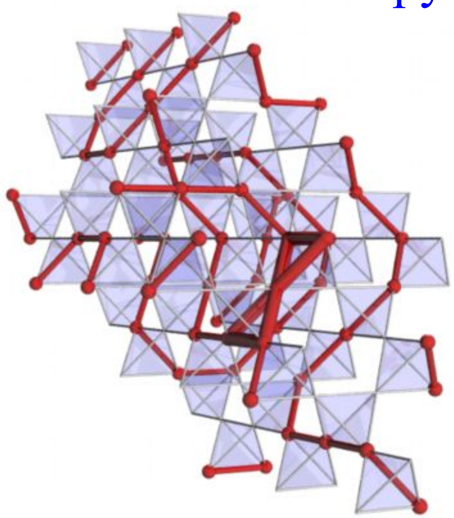
due to A processes

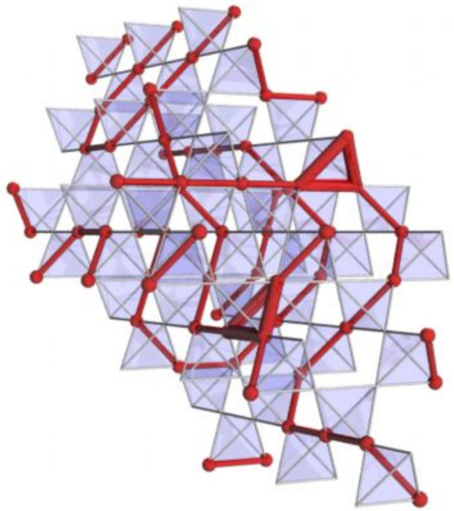
Addition of an electron → decay into two excitations with charge e/2 (backflow)



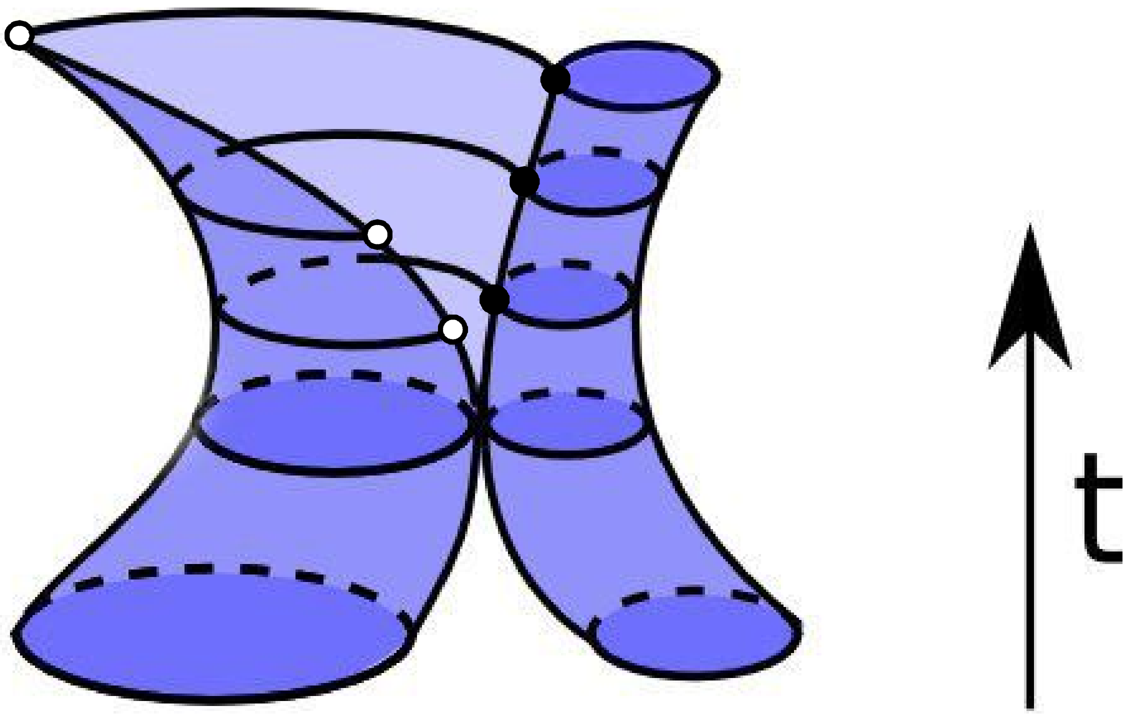


pyrochlore lattice:





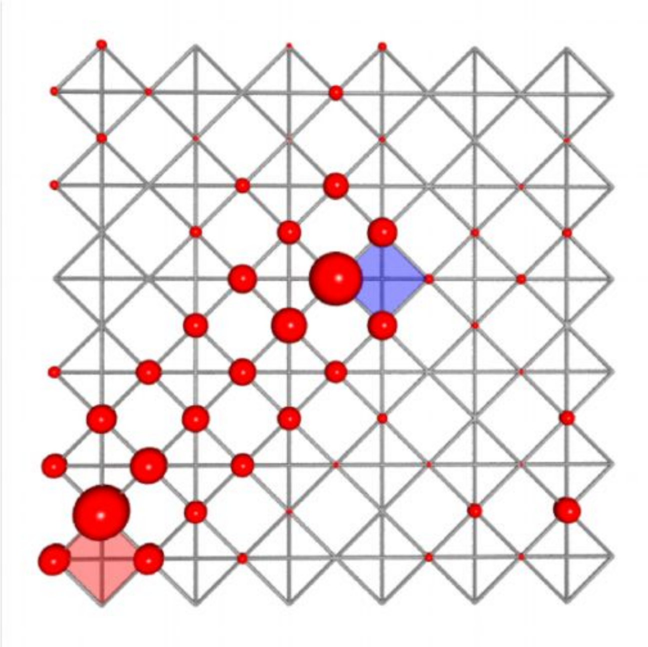
breaking of a loop → **creation of a pair** $+\frac{e}{2}, -\frac{e}{2}$



Confinement of fractional charges

change in kinetic energy in the presence of two fixed charges e/2 and –e/2

site *i*: $$\varepsilon_{\mathrm{i}} = -\frac{1}{6}\sum_{\hexagon/\mathrm{i}\varepsilon\hexagon}\left\langle \bar{\psi}_0(0,\mathbf{r})\middle|\mathrm{H}_{\mathrm{eff}}\middle|\bar{\psi}_0(0,\mathbf{r})\right\rangle$$



constant confining force

reason: vacuum fluctuations are reduced in the vicinity of the string

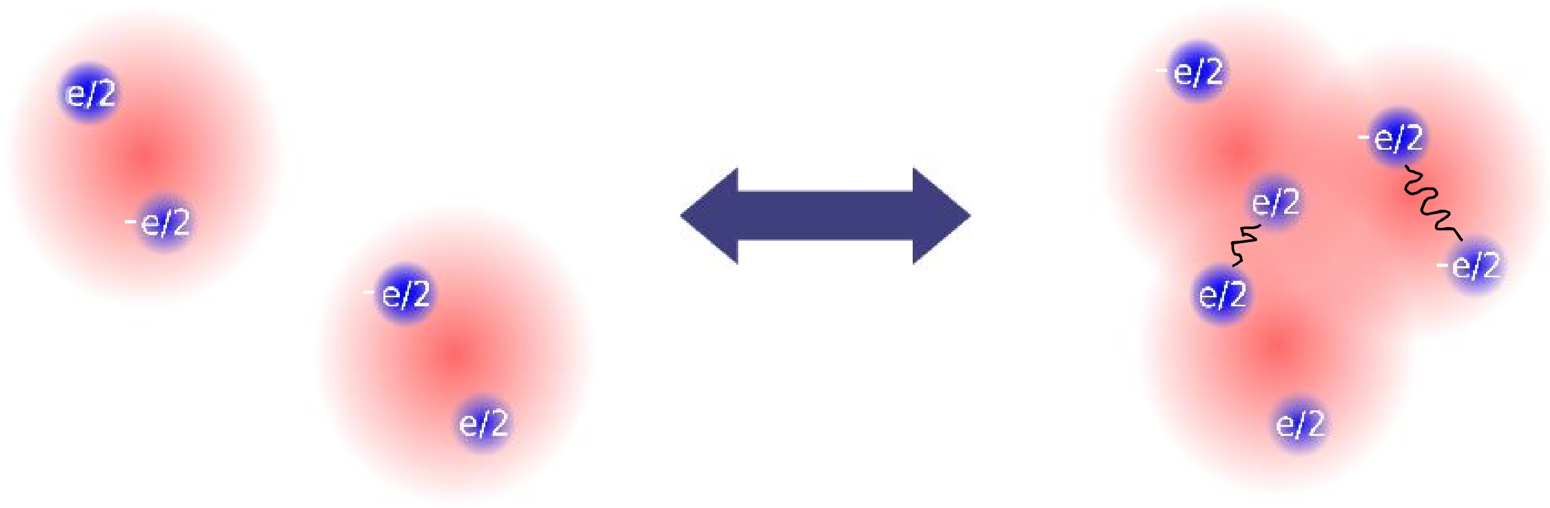
numerics: $\Delta\varepsilon_{\mathrm{kin}} \;;\; 0.2\,\mathrm{g}\cdot\mathrm{r}$ r = units of a

$\mathrm{r} > \mathrm{r_c} \;;\; 0.4\left(\mathrm{V}/\mathrm{t}\right)^3$ ⟹ $\left(\frac{\mathrm{e}}{2}, -\frac{\mathrm{e}}{2}\right)$ pair production

since $\Delta\varepsilon_{\mathrm{kin}} > \mathrm{V}$

- with increasing number of created pairs → formation of particle and antiparticle pairs e/2, e/2 and –e/2, -e/2

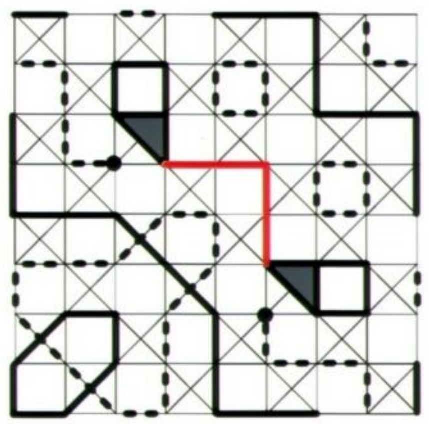
- since $g = \frac{12t^3}{V^2} \ll t$ → huge (extended) quasiparticles e.g., $d \sim 100a$



→ eventually transition to e/2, -e/2 plasma

Perspectives:

- Inclusion of spin:



additional $H_W = J \sum_{\langle ij \rangle} \mathbf{S}_i \mathbf{S}_j$

spin is highly nonlocal

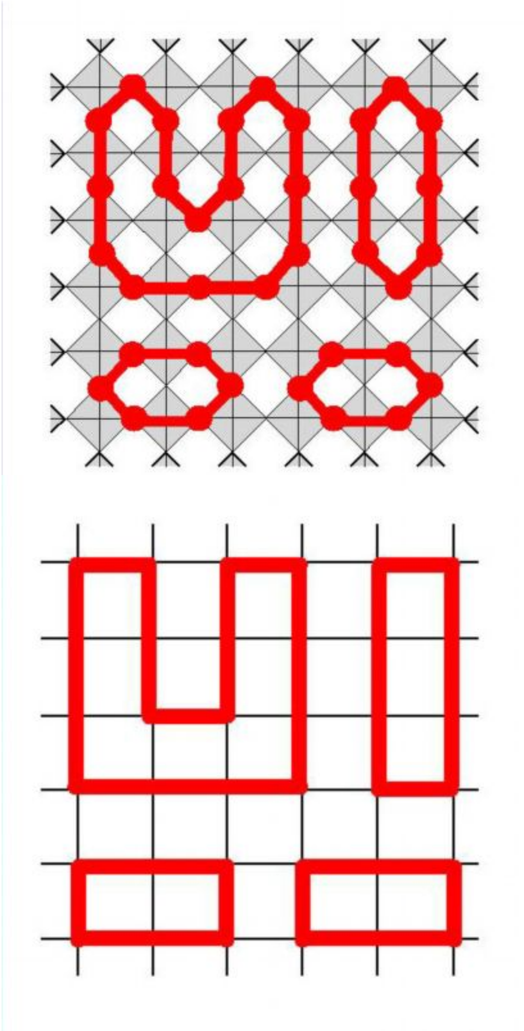
carried by „gluon“

- Statistics of e/2 charges

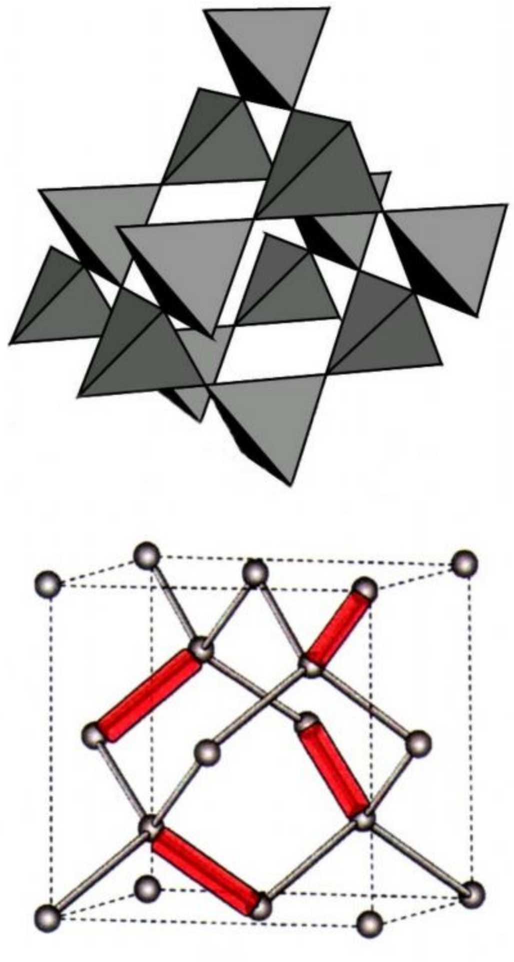
- 3D pyrochlore lattice: U(1) gauge theory allows for deconfined charges

Medial lattice

checkerboard



pyrochlore



U(1)-liquid phase in 3D quantum dimer model

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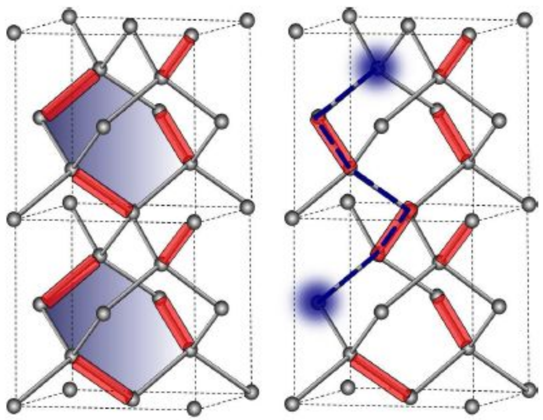
pyrochlore lattice ⟹ diamond lattice

$$H = -\sum_{\{\hexagon\}} \left(|\hexagon\rangle\langle\hexagon| + \mathrm{H.c.} \right) + \mu \sum_{\{\hexagon\}} \left(|\hexagon\rangle\langle\hexagon| + |\hexagon\rangle\langle\hexagon| \right)$$



(a) (b)

$\mu \to -\infty$: ground state maximizes number of flipable hexagons
⟹ R-state 8-fold degenerate

$\mu > 1$: ground states are many "isolated states"
⟹ no flippable hexagons

$\mu = 1$: RK point ⟹ all configurations have equal weight in ground state

order parameter: 6-dimensional irred. repres. of symmetry group
16-site unit cell

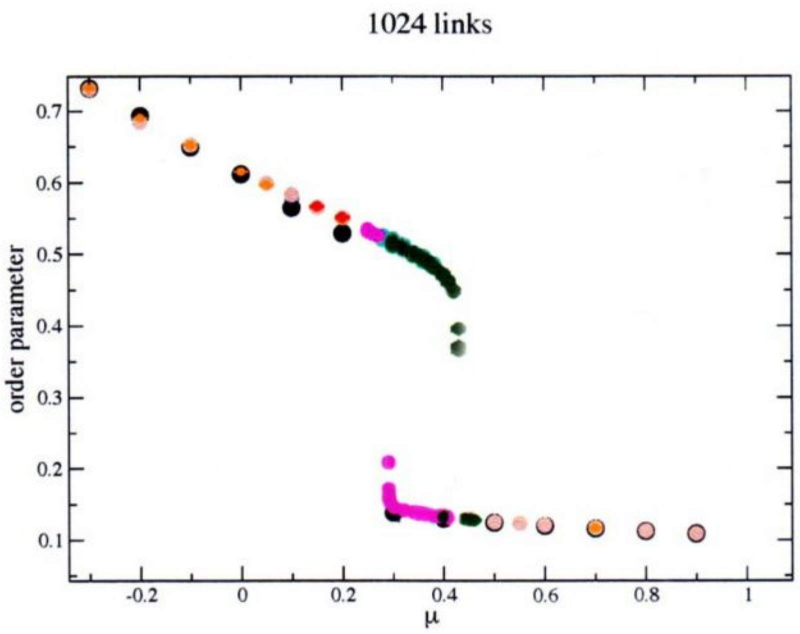
$$m_R = \langle\psi|\sqrt{\sum_{\eta=1}^{6}\left(\sum_{\xi=1}^{16} f_\eta^\xi \hat{n}^\xi\right)^2}\,|\psi\rangle$$

f = weight factors

Variational Monte Carlo

$$|\psi_{ext}\rangle \exp\left[\alpha N_{flip} + \beta m_R + \sum_{\langle ij\rangle} \gamma_{ij} n_i n_j\right]|\psi_{RK}\rangle$$

$$H = H_{RK} + (\mu-1)\sum_{\{\hexagon\}}\left(|\hexagon\rangle\langle\hexagon| + |\hexagon\rangle\langle\hexagon|\right)$$



quality of variational ground state checked

by GFMC liquid state for $0.4 < \mu < 1$