GENERALIZED HEISENBERG-EULER ENERGY AND TIME SCALES FOR STRONG ELECTRIC FIELD DEPLETION

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Plan of the talk:

- Introduction
- Mean-energy density
- Initial state as thermal equilibrium
- SU(3) chromoelectric field

QFT in an external background:

$$j^{\mu}A_{\mu} \to j^{\mu}\left(A_{\mu} + A_{\mu}^{\text{ext}}\right)$$

Strong-field QFT (quasiconstant field approximation):

$$\mathcal{L} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)},$$

Maxwell Lagrangian:

$$\mathcal{L}^{(0)} = \left(E^2 - B^2\right)/8\pi,$$

Heisenberg-Euler Lagrangian (vacuum polarization contribution):

$$\mathcal{L}^{(1)} = \int_0^{\infty} \frac{\exp\left(-im^2 s\right)}{8\pi^2 s}\left[e^2 EB \coth\left(eEs\right)\cot\left(eBs\right) - \frac{1}{s^2} - \frac{e^2}{3}\left(E^2 - B^2\right)\right] ds.$$

What is condition when quasiconstant field approximation is consistent, especially, for the case of an electriclike field?

Applications:

- Strong field experiments on SLAC and TESLA X-ray lasers,

- astrophysics of neutron and hot strange stars,

- graphene physics,

- initial state of quark-gluon plasma (chromoelectric flux tube model, color glass condensate).

Strong magnetic field ($B \gg m^2/e,\ E = 0$) yields

$$\mathcal{L}^{(1)} \approx -\left(\frac{\alpha}{3\pi} \ln \frac{eB}{m^2}\right) \mathcal{L}^{(0)} .$$

Restriction due to vacuum polarization:

$$B \ll F_{\text{max}},$$

e^+e^- uninteracted [Weisskopf, 1936; Ritus, 1975]:

$$F_{\text{max}} = \frac{m^2}{e} \exp\left(\frac{3\pi}{\alpha}\right) \approx \frac{m^2}{e} 10^{560} ,$$

e^+e^- interacted [Shabad and Usov, 2005]:

$$F'_{\text{max}} = \frac{m^2}{4e} \exp\left(\frac{\pi^{3/2}}{\sqrt{\alpha}} + 2 \times 0.577\right) \approx \frac{m^2}{e} 10^{28} .$$

HEL is related to the vacuum-to-vacuum transition amplitude:

$$c_v =< 0, out|0, in >= \exp\left(i \int dx \mathcal{L}^{(1)}\right) .$$

Strong electric field ($E \gg m^2/e$, $B = 0$) yields

$$\mathrm{Re}\,\mathcal{L}^{(1)} = -\frac{\alpha}{3\pi} \ln \frac{eE}{m^2} \mathcal{L}^{(0)} \,.$$

Is it relevant [Greenman and Rohrlich, 1973]

$$E \ll F_{\mathrm{max}} \approx \frac{m^2}{e} 10^{560} ?$$

Vacuum polarization (local, T-independent) + pair creation (global, T-dependent).

T-constant field regularization:

$$A_3(t) = \begin{cases} -Et_1\,, & t \in (-\infty, t_1)\,, \\ -Et\,, & t \in [t_1, t_2], \\ -Et_2\,, & t \in (t_2, +\infty)\,. \end{cases} ,$$

where T is sufficiently large

$$\left[1 + \frac{m^2}{eE}\right]^2 \ll eET^2. \qquad (1)$$

Mean-energy density with respect of the initial vacuum $|0, in\rangle$:

$$w = \frac{1}{2}\langle 0, in| \left[\psi(x)^{\dagger}, \mathcal{H}\psi(x)\right] |0, in\rangle\Big|_{x^0=t_2-0} ,$$

is independent of the spatial coordinates and time-dependent.

Representation via the in-in Green function:

$$\begin{aligned} w \;&=\; -\frac{1}{4}\Bigg[\lim_{t\to t'-0} \mathrm{tr}\left[\left(\partial_0 - \partial_0'\right) S_{in}(x,x')\right] \\ &\qquad + \lim_{t\to t'+0} \mathrm{tr}\left[\left(\partial_0 - \partial_0'\right) S_{in}(x,x')\right]\Bigg]\Bigg|_{\mathbf{x}=\mathbf{x}', x^0=t_2-0} , \end{aligned}$$

where

$$\begin{aligned} S_{in}(x,x') \;&=\; i\langle 0, in|T\psi(x)\bar{\psi}(x')|0, in\rangle \\ &=\; S^c(x,x') + S^p(x,x') \,. \end{aligned}$$

$S^c(x,x')$ is causal Green function

$$S^c(x,x') = i\frac{\langle 0, out|T\psi(x)\bar{\psi}(x')|0, in\rangle}{\langle 0, out|0, in\rangle} ,$$

where $|0, out\rangle$ is the final vacuum.

$S^c(x,x')$ can be represented as the Fock–Schwinger proper time integral:

$$S^c(x,x') = \int_0^\infty f(x,x',s)ds\,,$$

where $f(x,x',s)$ is the Fock–Schwinger kernel.

The function $S^p(x,x')$ represents contribution of created pairs and is linear functional of the differential mean number of pairs created from vacuum,

$$\aleph_{\mathbf{p},r} = \exp\left(-\pi\frac{m^2+\mathbf{p}_\perp^2}{eE}\right)\,.$$

Then contributions due to vacuum polarization and pair creation are separated:

$$w = \underset{\left({\text{vac. pol} \atop \text{loc.} \sim T^0}\right)}{w^c} + \underset{\left({\text{pair. cr} \atop \text{glob.} \sim T^2}\right)}{w^p}\,,$$

$$w^c = E\frac{\partial \mathrm{Re}\mathcal{L}^{(1)}}{\partial E} - \mathrm{Re}\mathcal{L}^{(1)} \approx -\left(\frac{\alpha}{3\pi}\ln\frac{eE}{m^2}\right)\mathcal{L}^{(0)},$$

Pair creation contribution:

$$w^p = \frac{1}{4\pi^3}\int_D d\mathbf{p} \sum_{r=\pm 1} \aleph_{\mathbf{p},r}\varepsilon_{\mathbf{p},r} \ ,$$
$$D \quad : \quad |p_3| < eET/2 \ ,$$

$\varepsilon_{\mathbf{p},r} = \sqrt{m^2 + \mathbf{p}_\perp^2 + (eET/2 - p_3)^2}$ - energy of out-particles, $\mathbf{p}_\perp = (p^1, p^2, 0)$.

T-leading term (being proportional to T^2):

$$w^p = eE\aleph T \ , \quad \aleph = \frac{e^2E^2T}{4\pi^3}\exp\left(-\pi\frac{m^2}{eE}\right) \ ,$$

$\aleph$ - the total number-density of pairs created.

From the condition

$$w^p \ll \mathcal{E}^{(0)} = E^2/8\pi$$

restriction from above:

$$eET^2 \ll \frac{\pi^2}{2e^2}\exp\left(\pi\frac{m^2}{eE}\right).$$

Initial state as thermal equilibrium at temperature θ :

$$w = \underset{(\sim T^0)}{w^c} + \underset{(\sim T^1)}{w^c_\theta} + \underset{(\sim T^2)}{\tau^p_\theta} ,$$

w^c - from vacuum polarization,

w^c_θ - from work of the field on particles from the many-particle initial state,

$\tau^p_\theta = w^p + w^p_\theta$ - energy density of pairs created from the many-particle initial state,

$$w^p = \frac{1}{4\pi^3} \int_D d\mathbf{p} \sum_{r=\pm 1} \aleph_{\mathbf{p},r} \varepsilon_{\mathbf{p},r}$$

$$w^p_\theta = -\frac{1}{4\pi^3} \int_D d\mathbf{p} \sum_{r=\pm 1} \aleph_{\mathbf{p},r} n_{\mathbf{p},r}(in)\, \varepsilon_{\mathbf{p},r} ,$$

$$n_{\mathbf{p},r}(in) = \left[\exp\left(\tilde{\varepsilon}_{\mathbf{p},r}/\theta\right) + 1\right]^{-1} ,$$

where $\tilde{\varepsilon}_{\mathbf{p},r} = \sqrt{m^2 + \mathbf{p}^2_\perp + (qET/2 + p_3)^2}$ is the energy of a free in-particle.

Fermions:

at low temperature, $\theta \ll eET$,

$$eET^2 \ll \frac{\pi^2}{2e^2} \exp\left(\pi \frac{m^2}{eE}\right).$$

at high temperature, $\theta \gg eET$,

$$\frac{(eE)^2 T^3}{\theta} \ll \frac{3\pi^2}{e^2} \exp\left(\pi \frac{m^2}{eE}\right).$$

Bosons ($J = 1$ for scalar particles, $J = 3$ for vector particles):

at low temperature, $\theta \ll eET$,

$$eET^2 \ll \frac{\pi^2}{Je^2} \exp\left(\pi \frac{m^2}{eE}\right),$$

at high temperature, $\theta \gg eET$,

$$\theta T \ln\left(\sqrt{eE}T\right) \ll \frac{\pi^2}{2Je^2} \exp\left(\pi \frac{m^2}{eE}\right).$$

Soft parton production by SU(3) chromo-electric field E^a $(a = 1, \ldots, 8)$.

The $\mathbf{p}_\perp$-distribution densities of gluons $n_{\mathbf{p}_\perp}^{gluon}$ and quarks $n_{p_\perp}^{quark}$ produced from vacuum:

$$n_{\mathbf{p}_\perp}^{gluon} = \frac{1}{4\pi^3} \sum_{j=1}^{3} Tq\tilde{E}_{(j)} \tilde{\aleph}_{\mathbf{p}}^{(j)},$$

$$n_{p_\perp}^{quark} = \frac{1}{4\pi^3} \sum_{j=1}^{3} TqE_{(j)} \aleph_{\mathbf{p}}^{(j)},$$

$$\tilde{\aleph}_{\mathbf{p}}^{(j)} = \exp\left(-\frac{\pi \mathbf{p}_\perp^2}{q\tilde{E}_{(j)}}\right),\ \aleph_{\mathbf{p}}^{(j)} = \exp\left(-\pi \frac{M^2 + \mathbf{p}_\perp^2}{qE_{(j)}}\right),$$

where $E_{(j)}$ are the eigenvalues of the matrix iT^aE^a for the fundamental representation of $SU(3)$; $\tilde{E}_{(j)}$ are the positive eigenvalues of the matrix $if^{abc}E^c$ for the adjoint representation of $SU(3)$;

$$\left|E_{(j)}\right| \leq \sqrt{C_1/3} \text{ and } \left|\tilde{E}_{(j)}\right| \leq \sqrt{C_1},$$

$C_1 = E^aE^a$ is Casimir invariants for $SU(3)$.

$$n_{p_\perp}^{gluon} \gg n_{p_\perp}^{quark},$$

then only the energy density of gluons created is important.

Total energy density of gluons created from vacuum:

$$w^p = \sum_{j=1}^{3} w^{p(j)}, \quad w^{p(j)} = \frac{1}{4\pi^3} \int_{D_{(j)}} d\mathbf{p} \aleph_{\mathbf{p}}^{(j)} \varepsilon_{\mathbf{p}}^{(j)} \ ,$$

from many-particle state at finite temperature:

$$\begin{aligned} w &= w^p + w_\theta^p, \quad w_\theta^p = \sum_{j=1}^{3} w_\theta^{(j)}, \\ w_\theta^{(j)} &= \frac{1}{4\pi^3} \int_{D_{(j)}} d\mathbf{p} \aleph_{\mathbf{p}}^{(j)} n_{\mathbf{p}}^{(j)}(in)\, \varepsilon_{\mathbf{p}}^{(j)}, \\ n_{\mathbf{p}}^{(j)}(in) &= \left[\exp\left(\tilde{\varepsilon}_{\mathbf{p}}/\theta\right) - 1\right]^{-1} . \end{aligned}$$

At low temperature, $\theta \ll q\sqrt{C_1}T$,

$$w \simeq w^p \lesssim q\sqrt{C_1}T\aleph^{gluon} , \ \aleph^{gluon} = \frac{3Tq^2C_1}{8\pi^3},$$

at high temperature, $\theta \gg q\sqrt{C_1}T$,

$$w \lesssim \frac{3\theta T q^2 C_1}{8\pi^3} \ln\left(q\sqrt{C_1}T^2\right).$$

Restrictions:

$$1 \ll q\sqrt{C_1}T^2,$$

At low temperature, $\theta \ll q\sqrt{C_1}T$,

$$q\sqrt{C_1}T^2 \ll \frac{\pi^2}{3q^2},$$

at high temperature, $\theta \gg q\sqrt{C_1}T$,

$$\theta T \ln\left(q\sqrt{C_1}T^2\right) \ll \frac{\pi^2}{3q^2}.$$

The above established consistency restrictions determine, in fact, the time scales from above of depletion of an electric field due to the backreaction.

See details in [S.P. Gavrilov and D.M. Gitman, arXiv:0805.2391; Phys. Rev. Lett. **101**, 130403 (2008); arXiv:0709.1828; Phys. Rev. D **78**, 045017 (2008).]