CAN ONE DETECT PASSAGE OF SMALL BLACK HOLE THROUGH THE EARTH?

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1. Introduction

Existence of black holes (BHs) is predicted by general relativity.
Heavy, $M \gtrsim 3M_\odot$, in binaries.
Superheavy, $M \sim 4 \times 10^7 M_\odot$, in some galaxies (including ours).
BHs lighter than $\lesssim 3M_\odot$ cannot arise from star compression.
But they could arise at the early stages of the Universe evolution from inhomogeneities of high matter density.
Too light primordial black holes have already evaporated and disappeared due to their thermal radiation (a subtle quantum-mechanical effect), with intensity proportional to $1/M^2$. The masses of the survivors should not be less than 5×10^{14} g. By common standards, such a mass is huge, but the size of survivor is tiny, $r_g \sim 10^{-12}$ cm.

These objects have never been observed, though their existence
does not contradict any known law of nature.
As a step to study the possibility
to detect the passage of such an object
through the Earth (or some other planet, or the Moon),
we discuss here the effects arising during this passage.

2. Coherent sound generation by supersonic black hole

At first, the dynamics of mechanical deformations and excitation of sound waves caused by the gravitational field of a primordial black hole passing through the Earth. We assume that velocity v of black hole is comparable to that of the Earth, which exceeds the speed of sound in the matter c_s.

The elastic gravitational scattering of a black hole in the matter results in supersonic sound radiation of the Cherenkov type with the total intensity

$$I_{\rm el} = 4\pi (GM)^2 \rho / v \, \ln \frac{k_1 c_s}{\omega_p} \,. \quad (1)$$

Here G is the Newton gravitational constant, M is the mass of black hole, ρ is the density of matter, $\omega_p = (4\pi G\rho)^{1/2}$, k_1 is the maximum momentum transfer at which the scattering remains elastic; we assume that $v \gg c_s$.

Qualitative explanation

Factor $(GM)^2\rho/v$ in $I_{\rm el}$ follows from physical and dimensional arguments.

Now on $\log$ in $I_{\rm el}$. Wave equation for gravitational potential is

$$\frac{1}{c_s^2}\ddot{\phi} - \Delta\phi - \frac{4\pi G\rho}{c_s^2}\phi = -4\pi GM\delta(\mathbf{r} - \mathbf{v}t)\,,$$

c_s is speed of sound, ρ is density; "plasma frequency" squared $\omega_p^2 = 4\pi G\rho$ enters with opposite sign since gravitating dust is unstable under density fluctuations, as distinct from quasineutral plasma which is stable under charge fluctuations. Scalar characteristic of matter deformation $\psi = -\delta\rho/\rho$ satisfies analogous wave equation with $\delta(\mathbf{r} - \mathbf{v}t)$ in rhs. $I_{\rm el}$ is quadratic in ψ.

Compare with electrodynamics where field strength F satisfies wave equation with $\partial_\mu \delta(\mathbf{r} - \mathbf{v}t)$ in rhs, and thus has extra ω as compared with ψ. F also enters radiation intensity quadratically. Frequency spectrum of common electromagnetic Cherenkov radiation is well known: $\omega d\omega$.

Then it is only natural that frequency spectrum of sound Cherenkov radiation is $d\omega/\omega$.

Back to the main problem.

For large momentum transfers, $k > k_1$, the matter can be considered as a collection of free particles. Then the scattering of the black hole reduces to that on a free particle. Thus obtained rate of inelastic energy loss by a black hole is

$$I_{\mathrm{inel}} = 4\pi (GM)^2 \rho / v \, \ln \frac{1}{k_1 a} \,, \tag{2}$$

where a is the typical interatomic distance in the matter.

Finally, the total rate of energy loss is

$$I_{\mathrm{tot}} = I_{\mathrm{el}} + I_{\mathrm{inel}} = 4\pi (GM)^2 \rho / v \, \ln \frac{c_s}{\omega_p \, a} \,. \tag{3}$$

With the accepted logarithmic accuracy, this total rate is independent of the critical momentum transfer k_1. On the other hand, for any reasonable choice of k_1, the elastic energy loss dominates strongly, $I_{\rm el} \gg I_{\rm inel}$. It is worth mentioning also that the logarithm in (3) is really large, about 35!

To estimate the energy ΔE released by a black hole passing through the Earth, this rate should be multiplied by the time of the passage, $\tau = L/v$. For numerical estimates we assume that the equilibrium density of matter is $\rho = 6$ g/cm^3, the path L is about the Earth diameter, $L \sim 10^4$ km, and the velocity of black hole is $v \sim 30$ km/s. Then, for a black hole with mass $M \sim 10^{15}$ g this energy loss constitutes about

$$\Delta E \sim 4 \times 10^9 \, {\rm J}\,. \tag{4}$$

This energy is much smaller than that released at the explosion of a 10 kiloton atomic bomb

$$\boldsymbol{\Delta E}_{\mathrm{bomb}} \sim \mathbf{5 \times 10^{13}}\ \mathrm{J}\,. \tag{5}$$

Besides, when comparing the energy released by a black hole (not only (4), but other possible contributions to it as well) with the energy of an atomic bomb, one should keep in mind that the source of $\boldsymbol{\Delta E}_{\mathrm{bomb}}$ is practically point-like, while $\boldsymbol{\Delta E}$ is spread along a path $\boldsymbol{L} \sim \mathbf{10^4}$ km. Then, the energy released by a black hole is extended not only in space, but in time: it takes several minutes for the black hole to cross the Earth, but the release of energy in an atomic bomb or in an earthquake happens in much shorter time intervals.

Due to this extension of the effect both in space and time, it is much more difficult to detect the passage of a small black hole.

To discover a mini-black hole passing through the Earth, one has to study seismic vibrations induced by this passage. The sensitivity of appropriate seismic detectors is confined to the frequencies in the interval around $\omega_{min} \sim 0.1$ Hz and $\omega_{max} \sim 10$ Hz. As to the frequency distribution of the acoustic Cherenkov radiation, it is

$$dI_{\rm el} \, = \, 4\pi (GM)^2 (\rho/v) \, \frac{d\omega}{\omega} \, . \tag{6}$$

Thus, the energy of the vibrations excited in the frequency interval $\omega_{min} \div \omega_{max}$ equals

$$\Delta E^{\omega} = 4\pi (GM)^2 L\rho/v^2 \, \ln \frac{\omega_{max}}{\omega_{min}} \, . \tag{7}$$

For the discussed frequency interval, 0.1 ÷ 10 Hz, it is numerically

$$\boldsymbol{\Delta E^{\omega} \sim 5 \times 10^{8}}\ \mathrm{J}\,, \tag{8}$$

or about 1/10 of the total energy (4).

Similar results hold for the energy losses of a mini-black hole in water, which

could be of interest for the underwater cosmic-ray acoustic detectors.

The underwater path L is much less than the Earth diameter, and the density of the medium changes here from $\rho \simeq 6$ g/cm^3 to $\rho = 1$ g/cm^3.

Though the typical frequencies propagating in water, both ω_{min} and ω_{max}, are much higher than those in the Earth, the value of $\ln \omega_{max}/\omega_{min}$ is about the same, to our accuracy.

3. Conversion of black hole radiation into sound waves

One more source of the energy transfer from a light black hole to the matter is the black hole radiation. Here we have to consider the emission of γ and $e^{\pm}$ only (but not gravitons and neutrinos). Under the same assumptions ($M \sim 10^{15}$ g, $L \sim 10^{4}$ km, and $v \sim 30$ km/s) we arrive at the following estimate for the total radiation loss of such black hole:

$$\Delta E_{\mathrm{rad}} \sim 1.5 \times 10^{12}\ \mathrm{J}\,. \tag{9}$$

Leading mechanism for conversion of this radiation into sound waves:
Radiation absorbed by matter increases the temperature along the path $\longrightarrow$
inhomogeneous and non-stationary thermal expansion of the matter $\longrightarrow$
emission of acoustic waves.

Effect is described by the following equation for pressure p
(P.G. Westervelt, R.S. Larson, J. Acous. Soc. Amer. **54**, 121 (1973)):

$$\frac{1}{c_s^2}\ddot{p} - \Delta p = \frac{\beta}{C}\dot{W}\,. \tag{10}$$

Here $\beta = -1/\rho\ (\partial\rho/\partial T)_p$ is coefficient of thermal expansion, C is specific heat, W is power density. Black hole can be treated as point-like source of radiation with intensity I, so that $W = I\delta(\mathbf{r} - \mathbf{v}t)$.
Spectral intensity of thus induced sound waves is

$$\frac{dE_m}{dt} = \left(\frac{\beta I}{C}\right)^2 \frac{\omega\, d\omega}{4\pi v\rho}\,. \tag{11}$$

Again qualitative explanation

Form of wave equation (10) is dictated by physical and dimensional arguments (up to a numerical factor in rhs).

This sound radiation occurs if $v > c_s$, it is also a sort of Cherenkov effect.

With time derivative in rhs of (10), and with quadratic dependence of emitted energy on p, the frequency spectrum should be (as for common Cherenkov radiation) $\omega \, d\omega$. Then, structure of result (11) is also dictated by physical and dimensional arguments.

The maximum value $\boldsymbol{\Omega}$ for the emitted frequencies is

$$\boldsymbol{\Omega \sim c_s/r_0\,;} \tag{12}$$

here $\boldsymbol{r_0}$ is the typical absorption length for γ and $\boldsymbol{e^{\pm}}$ emitted by the black hole (about 3 cm), so that $\boldsymbol{\Omega}$ is much larger then the frequencies of interest.

The total energy radiated at frequency $\boldsymbol{\omega}$, during the passage of a black hole through the Earth, is

$$\boldsymbol{\frac{dE_m}{d\omega} = \left(\frac{\beta \Delta E_{\mathrm{rad}}}{C}\right)^2 \frac{\omega}{4\pi L \rho}\,;\;\; \Delta E_{\mathrm{rad}} = I\,L/v.} \tag{13}$$

Overall energy of the seismic waves generated by the black hole radiation, released in the frequencies $\omega < \omega_{\max}$ is

$$E_m^{\omega} = \left(\frac{\beta \Delta E_{\mathrm{rad}}}{C}\right)^2 \frac{\omega_{\max}^2}{8\pi L \rho}. \tag{14}$$

With $C = 1\,\mathrm{J\,g^{-1}\,K^{-1}}$, $\beta = 0.5 \cdot 10^{-4}\,\mathrm{K^{-1}}$, and $\omega_{\max} = 10$ Hz, we obtain $E_m^{\omega} \sim 1\,\mathrm{J}$.

This effect is much less than that of Cherenkov sound radiation.

Situation changes essentially for the underwater effects.

Here the admissible frequencies extend to about $\omega_{\max} = 30$ kHz
(which is already on the order of Ω), and thus

conversion of the black hole radiation into sound waves
is about as effective as the Cherenkov sound radiation.

Radiation damage

Though effects of radiation damage contribute negligibly into seismic signal, they can create quite a distinct pattern in crystalline material.
The dose deposited is estimated as

$$\frac{\boldsymbol{\Delta E}_{\mathrm{rad}}}{\boldsymbol{\rho L r_0^2}} \sim \mathbf{10^5}\ \mathrm{Gy} \qquad (\,\mathbf{1}\mathrm{Gy}\ (\mathrm{Gray}) = \mathbf{1}\ \mathrm{J/kg}\,)\,. \tag{15}$$

It creates a long tube of heavily radiatively damaged material, which could stay recognizable for geological time.