The Role of Sum Rules in the Discovery of the Standard Model

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(This talk overlaps with "Inelastic Sum Rules", talk at the Sid Drell Symposium, SLAC, Stanford, California, July 31st, 1998, arXiv:hep-ph/9812301v)

Attitudes in the 1960s

Field Theory

Chew (1961): "*I believe the conventional association of fields with strongly interacting particles to be empty. I do not have firm convictions about leptons or photons ... field theory ...., like an old soldier, is destined not to die but just to fade away"".* This was the fashionable view.

Proton accelerators believed to hold the keys

- 1969 European Particle Physics Conference (Lund) ~ 610 participants
- 1969 International Symposium on Electron and Photon Interactions at High Energy ~ 240 participants

No deep inelastic data until 1968

1965

Adler Sum Rule*, in modern notation:

$$\int_0^1 (F_2^{\upsilon n}(x,q^2) - F_2^{\upsilon p}(x,q^2))\frac{dx}{2} = 1$$

$$\Rightarrow \lim E\to\infty: \frac{d\sigma^{\upsilon n}}{dq^2} - \frac{d\sigma^{\upsilon p}}{dq^2} = \frac{G_F^2}{\pi}$$

Parton interpretation: $n_u + n_{\bar{d}} - n_d - n_{\bar{u}} = 1$ ***but** sum rule is exact (derivation on next slide)*

Highly suggestive of point-like behaviour – Bjorken (1967)

Data (BEBC, 1985, corrected for later value of σ^{ν}) $q^2 = 1 – 40$ GeV^2: 1.08 +/- 0.08 +/- 0.18

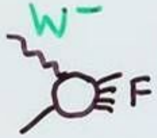
* As $q^2 \to 0$, using Adler's forward neutrino theorem and PCAC, reduces to the Adler-Weissberger relation (1964) between g_A and $\sigma_{\pi N}$

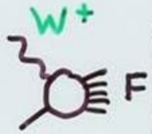
Derivation of Adler Sum Rule

- true at all Q^2

$$W_{\mu\nu} = \sum \int \frac{d^4x}{4\pi} e^{iq\cdot x} \langle P | [J_\mu^+(x), J_\nu^-(0)] | P \rangle$$

$$= \sum_F \Big[\Big| \text{(diagram 1)} \Big|^2_{\nu>0} - \Big| \text{(diagram 2)} \Big|^2_{\nu<0} \Big]$$





$$= -g_{\mu\nu} W_1 + \frac{P_\mu P_\nu}{M^2} W_2 + i\frac{\epsilon_{\mu\nu\alpha\beta} P^\alpha q^\beta}{2M^2} W_3 + \cdots$$

$$\lim_{|\vec{P}|\to\infty} \int dq_0 W_{00} \text{ in frame } \vec{q}\cdot\vec{P} = 0$$

so $dq_0 = \dfrac{d\nu}{\sqrt{\vec{P}^2 + M^2}}$, $q^2 = \dfrac{\nu^2}{\vec{P}^2 + M^2} - \vec{q}^{\,2} \longrightarrow -\vec{q}^{\,2}$

$$\stackrel{?}{=} \int_0^\infty \left(W_2^{\bar{\nu}}(\nu, q^2) - W_2^{\nu}(\nu, q^2) \right) d\nu = 2M^2.$$

1966

Bjorken's polarisation sum rule *(derivation on next page)*

$$\int (g_1^p(x,q^2) - g_1^n(x,q^2))dx = \frac{1}{6}\left|\frac{g_A}{g_V}\right|(1 - \frac{\alpha_s}{\pi} + ...$$

Described by Bj as "worthless" (although now tested) but used to derive inequality for unpolarised scattering: *"inelastic scattering large….comparable to scattering off point-like charges"*

QCD corrections now known to order α_s^3

Data: g_A/g_V = 1.20 +/- 0.08/0.07 +/- 0.12 +/- 0.10/-0.04

PDG: 1.2601+/- 0.0025

Same techniques:

$\sigma_{e^+e^-} \sim 1/E^2$ Bj "The idea that the total hadronic yield from colliding [e^+e^-] beams should be approximately the same as the $\mu^+\mu^-$ yield is folklore*" ** B Richter, private communication*

$\sigma^{\nu N} \sim E$

Bjorken, Johnson, Low Cornwall, Norton- large Q^2 Sum Rules

Spin less currents :

$$T = i\int \Theta(x_0)\, e^{iq\cdot x} \langle p|[J^{+}(x), J(0)]|P\rangle\, d^4x$$

+ polynomial in q_0

$$\xrightarrow{q_0 \to i\infty} \sum_{n=0}^{\infty} \left(\frac{i}{q_0}\right)^{n+1} \int e^{-i\vec{q}\cdot\vec{x}} \langle p|\left[\frac{\partial^n J^{+}(\vec{x},0)}{\partial t^n}, J(0)\right]|P\rangle$$

+ poly. in q_0

$$= T(0, q^2) + 2\nu \int_{-\infty}^{+\infty} \frac{W(\nu', q^2)}{\nu'(\nu'-\nu)}\, d\nu'$$

- limit $q_0 \to i\infty$ here also, frame $\vec{q} = 0$
- equate coeffs. of $(1/q_0)^n$
- divide by $(p_0)^n$ + let $p_0 \to \infty$

eg Neutrino case :

With $F_1^{\pm}(\omega) = \lim_{q^2\to\infty} \left(W_1^{\nu}(\omega, q^2) \pm W_1^{\bar{\nu}}(\omega, q^2)\right)$

↑ assumed to exist

$$2\int F_1^{\pm} \frac{d\omega}{\omega^{n+2}} = \lim_{p_0\to\infty} i\left(\frac{i}{2p_0}\right)^{n+1} \int \langle p|\left[\frac{\partial^n J^{+}(\vec{x},0)}{\partial t^n}, J^{-}(0)\right]|P\rangle\, d^3x$$

+ if n even

− if n odd

Bjorken spin sum rule ~ n = 0

Higher moments ~ assumptions about Hamiltonian

1967

Bjorken - SLAC Conference + Varenna Lectures

- Interpretation of Adler sum rule in terms of incoherent scattering off point-like constituents - next slide: partons (before Feyman), but no explicit statement of scaling
- **Gottfried:** ‘breathtakingly crude’ sum rule gave ~ parton charges (but no data until later) :

$$\sum_{i,j} Q_i Q_j = \sum_i Q_i^2 + \sum_{i \neq j} Q_i Q_j$$

Correlations vanish for p and n, so using closure approximation get sum rule (“Idiotic, or some truth?”: want “a derivation that a well educated person could believe”)

Bjorken (SLAC conference) re-wrote/generalised - “diffractive contributions should presumably be ignored”. In modern notation

$$\int \left(F_2^{ep}(x,q^2) - F_2^{en}(x,q^2) \right) dx = \frac{1}{3}$$

Would now put $\frac{1}{3} + \frac{2}{3}\left(n_{\bar{u}} - n_{\bar{d}}\right)$

Data (came later) **< 1/3** (today 0.240 +/- 0.016). Reasonable:

$$\left(u\bar{u}\right) < \left(d\bar{d}\right) \ \text{Pauli} \ + \left(p \leftrightarrow \pi^+ n + \pi^0 p\right) \Rightarrow \bar{d} > \bar{u}$$

Bjorken (1967 SLAC Conference)

'This result would also be true were the nucleon a point-like object, because the derivation is a general derivation. Therefore the difference of these two cross sections is a point-like cross section, and it is big'.

He goes on to suggest an interpretation, as follows: *'We assume that the nucleon is built of some kind of point-like constituents which could be seen if you could really look at it instantaneously in time ... If we go to very large energy and large* q^2*... we can expect that the scattering will be incoherent from these point-like constituents. Suppose ... these point-like constituents had isospin one-half ... what the sum rule says is simply* [N ↑] − [N ↓] = 1 *for any configuration of constituents in the proton. This gives a very simple-minded picture of this process which may look a little better if you really look at it, say, in the centre-of-mass of the lepton and the incoming photon. In this frame the proton is ... contracted into a very thin pancake and the lepton scatters essentially instantaneously in time from it in the high energy limit. Furthermore the proper motion of any of the constituents inside the hadron is slowed down by time deletion. Provided one doesn't observe too carefully the final energy of the lepton to avoid trouble with the uncertainty principle, this process looks qualitatively like a good measurement of the instantaneous distribution of matter or charge inside the nucleon'*

1968

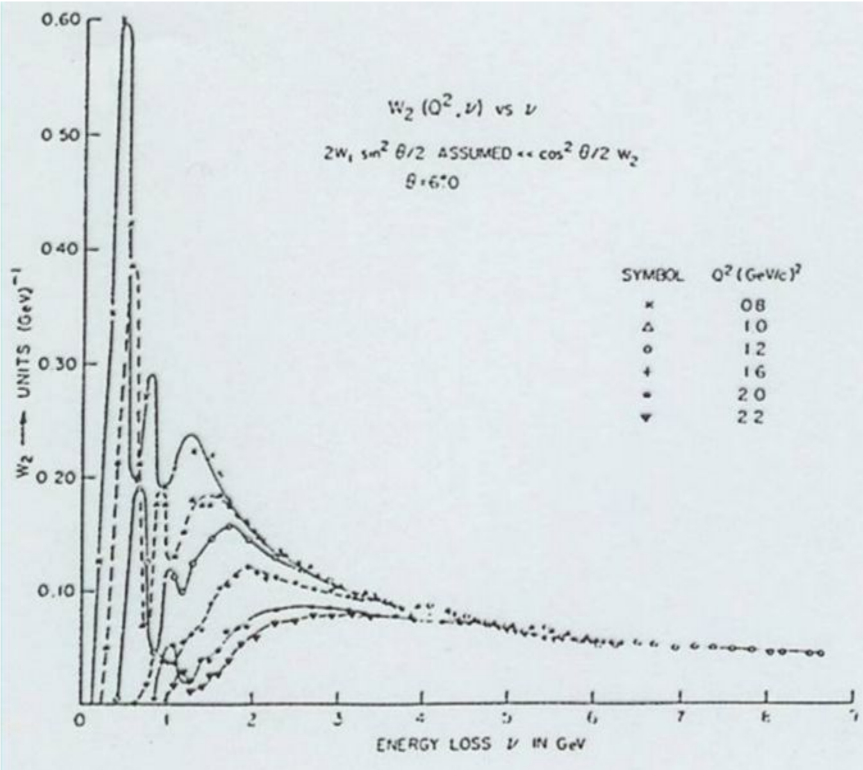
- **Bjorken's "derivation" of scaling (published 1989)**
  But: "a more physical interpretation of what is going on is, without question, needed"! *Having had partons without scaling, Bj then had scaling without partons!*

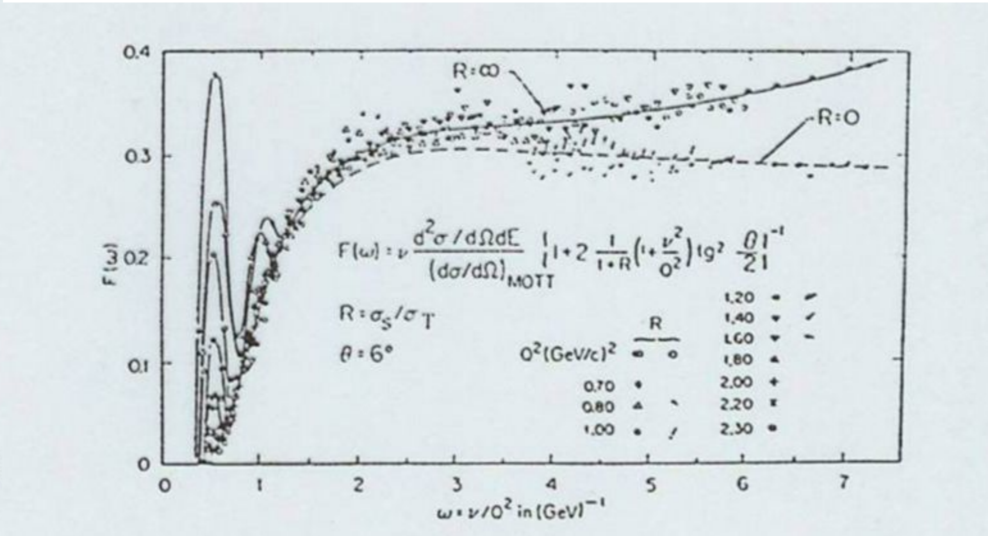
- **First data deep-inelastic data from SLAC** (next slide)
  presented by Panofsky at the Vienna conference:

  These cross sections *'are very large and decrease much more slowly with momentum transfer than the elastic scattering cross sections and the cross sections of the specific resonance states ... therefore theoretical speculations are focused on the possibility that these data might give evidence on the behaviour of point-like, charged structures within the nucleon ... The apparent success of the parametrization of the cross sections in the variable* ν/q^2 *in addition to the large cross section itself is at least indicative that point-like interactions are becoming involved'*.

  (Panofsky also discussed the experimental status of a sum rule derived by 'Godfrey', i.e. Gottfried)

First Deep Inelastic Data from SLAC





1969

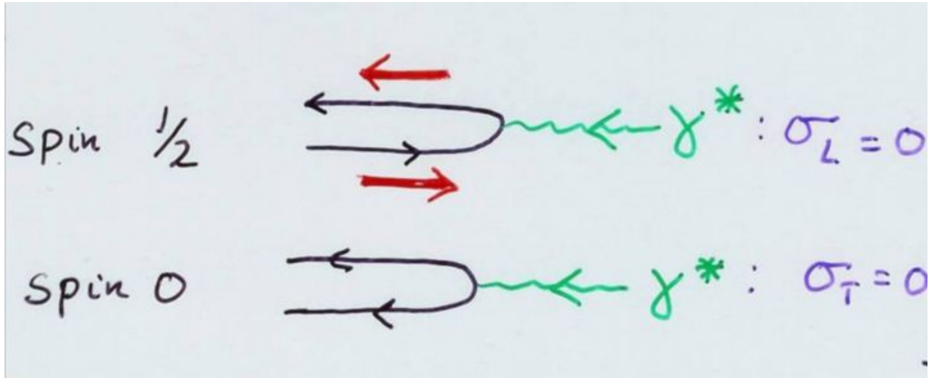
Partons (basic idea Bjorken 1967)

- Feynman – unpublished
- Bjorken & Pashcos – explicit model (no gluons)
- Drell, Levy, Yan – πN field theory with transverse momentum cut-off in infinite momentum frame –laboratory to identify processes to which parton ideas might apply

Callan-Gross

Related $\int F_2 \frac{dx}{x}$ and $\int F_1 \frac{dx}{x^2}$ to $[\dot{J}, J]$

Model dependent: $\frac{\sigma_L}{\sigma_T} = 0$ quark model, $= \infty$ algebra of fileds



1969

- **Failure of scaling in perturbation theory**

Adler & Tung, Jackiw & Preparata

- **G LI S** $$\int F_3^{\nu N} \frac{dx}{2x} = 3(1 - \frac{\alpha_s}{\pi} + \ldots$$

Reaction to powers of ln(q^2) found in perturbation theory “No reason to believe field theory relevant – contradicted by experiment”

QCD corrections known to order α_s^3

Used to measure $\alpha_s(s)$:

Data at Q^3 = 3 correspond to $\alpha_s(M_Z)$ = 0.118 +/- 0.011

(PDG give 0.1176± 0.002)

- **First data (September) on σ_L/σ_T = 0.2 +/- 0.2**

Test of G Ll S Sum Rule

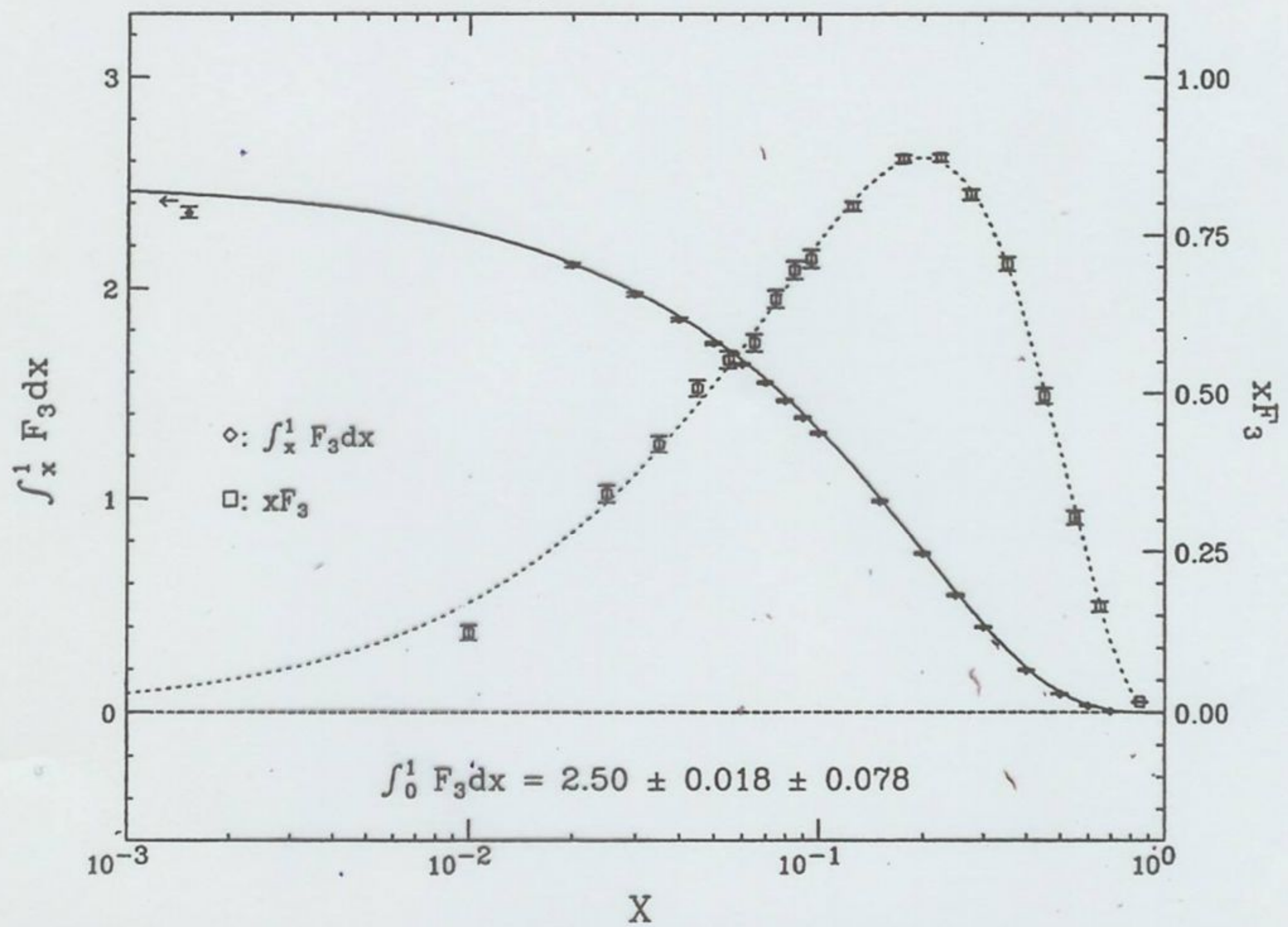
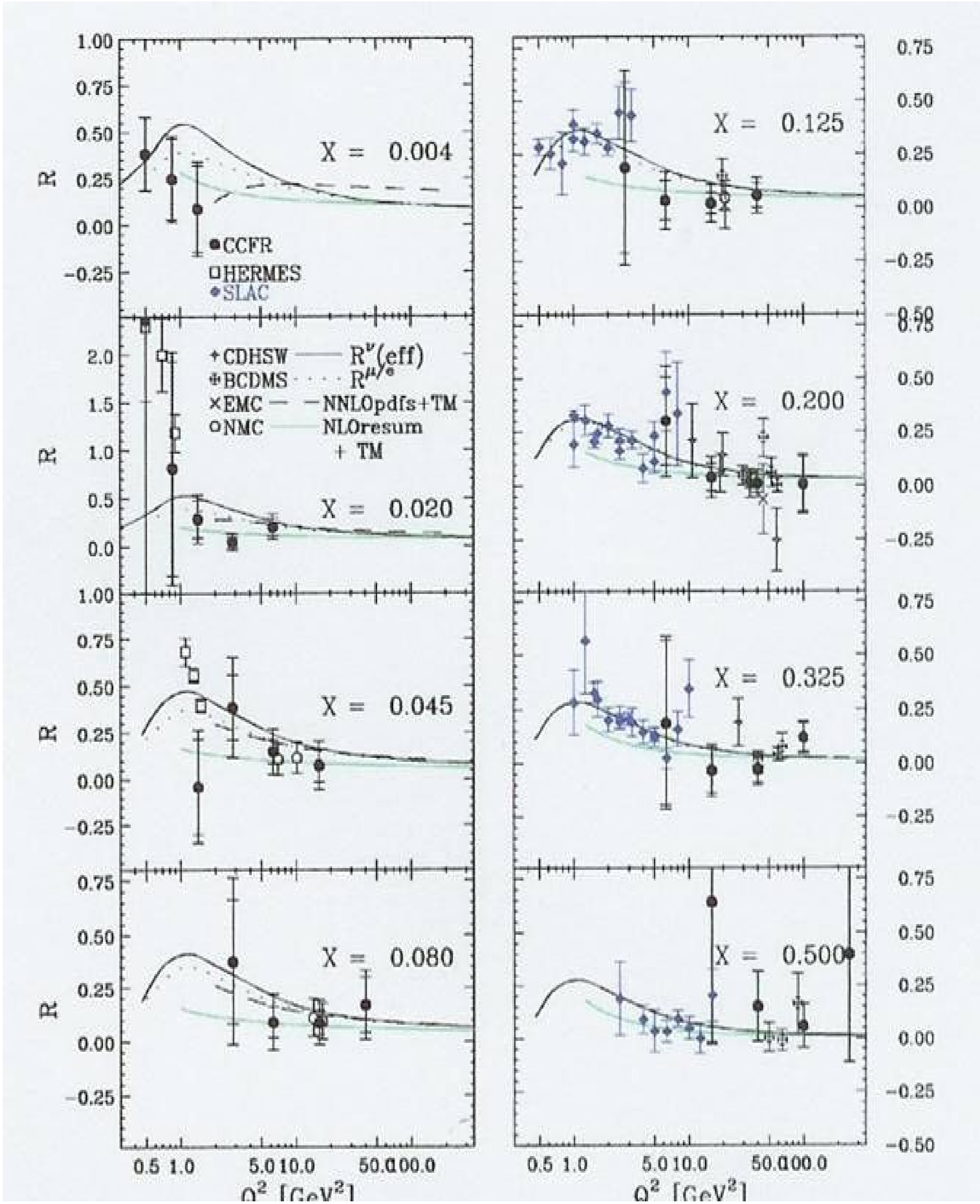


Figure 5: *CCFR test of the Gross-Llewellyn Smith sum rule. The dashed line indicates the parametrisation of the data used to evaluate the integral and the solid line illustrates the convergence of the integral at small x.*

$R = \sigma_L/\sigma_T$



1970

Many ideas still on market – diffractive model, Harari model, generalised vector meson dominance, Veneziano-like model…as well as quarks/partons

C LI S: results true in all quark-parton models (rederived formally):
$$F_3^{\nu p} - F_3^{\nu n} = 12\,(F_1^{ep} - F_1^{en})$$

$$F_2^{ep} + F_2^{en} \geq \frac{5}{18}(F_2^{\nu p} + F_2^{\nu n}) \quad \text{equality if no strange quarks}$$

$$\text{but equivalent to} \quad \frac{\sigma_{\gamma^*[I=o]}}{\sigma_{\gamma^*[I=1]}} = \frac{1}{9}$$

known to work for real photons: explained by VDM – so, nothing to do with quarks!?

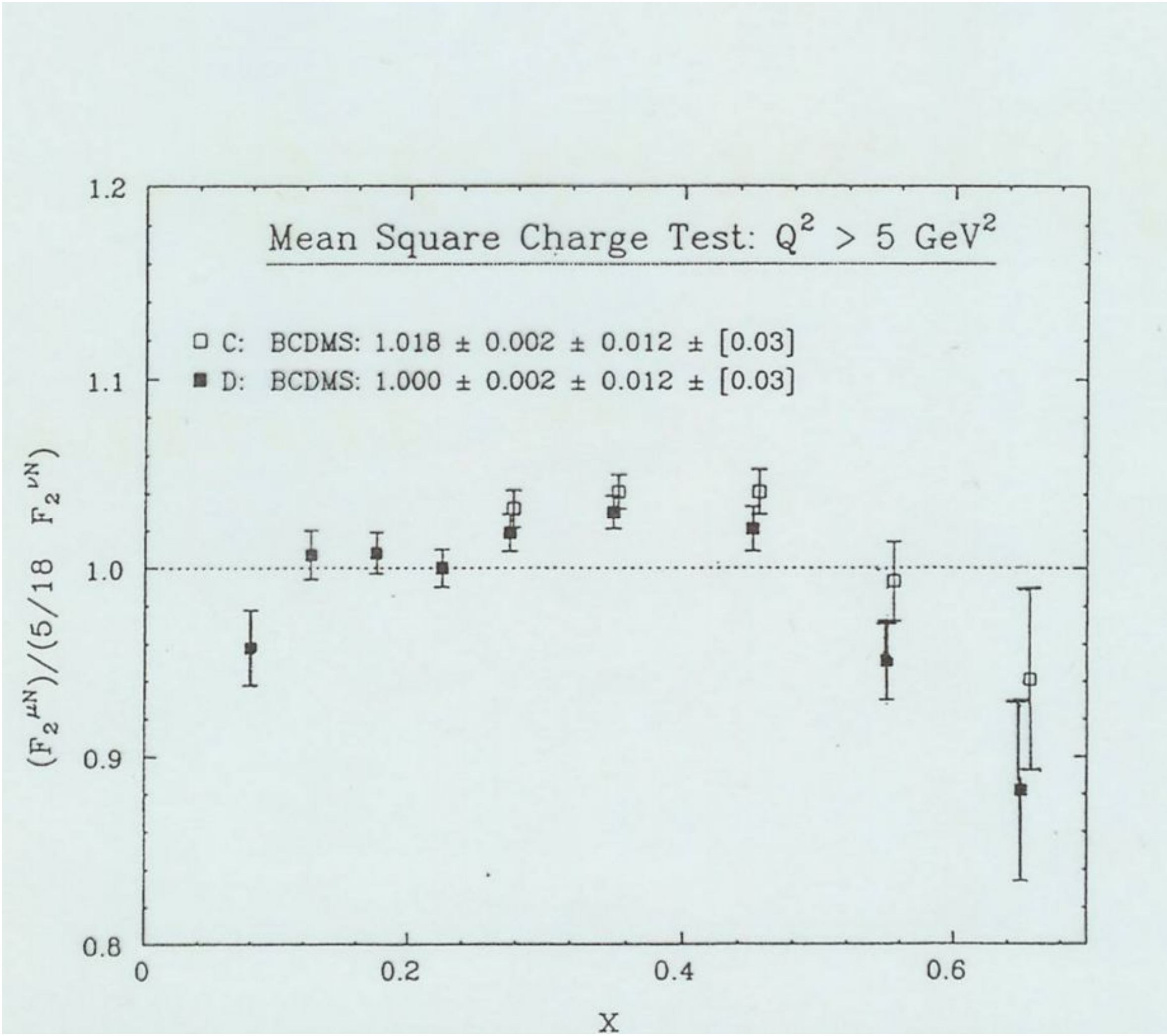
$$\int F_2^{ep} dx = \sum_i Q_i^2 < x_i >$$

Too big with only quarks: 1/3 with 3 quarks, $\geq$ 2/9 any 3q + sea model. Data 0.18

"easily reduced by adding a background of neutral constituents (which could be responsible for binding quarks)" attacked as not in the spirit of the quark model!

Test of Quark Charges

(assuming no strange, charm…quarks)



1971 ('t Hooft - gauge theories renormalisable)

Momentum sum rule (C LI S)

$$\varepsilon(\text{momentum in gluons}) = 1 + \int \left(\frac{3}{4}[F_2^{\nu p} + F_2^{\nu n}] - \frac{9}{2}[F_2^{ep} + F_2^{en}] \right)$$

Data, 1971 $\geq$ 0.52 +/- 0.38

Latest fits:	0.38	0.44	0.47	0.48
Q^2	2	30	2000	2000,000

1972

- Gribov & Lipatov sum leading logs in Abelian gauge theory

Reproduced by Christ, Hasslacher and Mueller using operator product expansion

- $$\frac{\sigma^{\bar{\nu}}}{\sigma^{\nu}} \approx \frac{1}{3}$$

1973 (neutral currents)

- **Asymptotic freedom, QCD**
- Gargamelle $\frac{d\sigma^{\nu}}{dy}$

BUT high y anomaly from HPWF

1974

- First calculations of QCD scaling violations, corrections to sum rules

- $\sigma_{e^{+}e^{-}} \rightarrow \text{constant} ??!!$ - see next slide

also tri-muons, high y anomaly..

- November: J/ψ

FIRST SLAC e^+e^- Data

At the 1974 London conference, in a session with 61 theoretical contributions, **B Richter** (rapporteur):

'the data contradict both the simple quark–parton model and the Bjorken scaling hypothesis".

Commenting on his own *"favourite models involving new lepton–hadron interactions"* he was *"struck ... by similar features seen in hadronic interactions"*, on first seeing the data, he had suggested that a kind of *"no photon annihilation"* was involved and had found that he was in distinguished company (Pati and Salam).

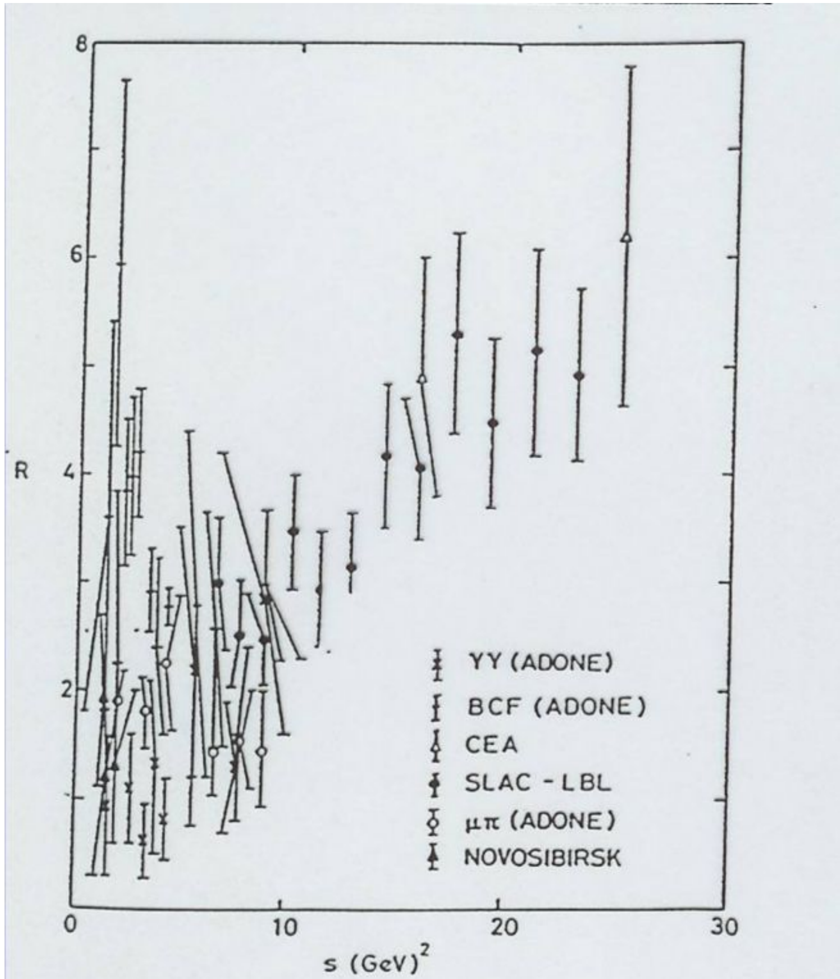


Fig. 4. All measurements of the ratio of σ_{TOT} to $\sigma_{\mu\mu}$ vs S, for $S \geq 1.5$ (GeV)^2.

1975

First evidence of scaling violations (next slide)

1976

Charm….

1977

Perturbative QCD

Subsequent 30 years

- overwhelming evidence for QCD: now mainly of interest as background, rather than signal
- lot of work on higher order corrections to sum rules, higher twist, and relations between coefficients in different cases (Crewther relation etc) – see particularly work by A Kataev et al
- sum rules now used as constraints on parton distributions, or ways to measure α_s

First Evidence of Scaling Violations
(also from SLAC but at lower Q^2)

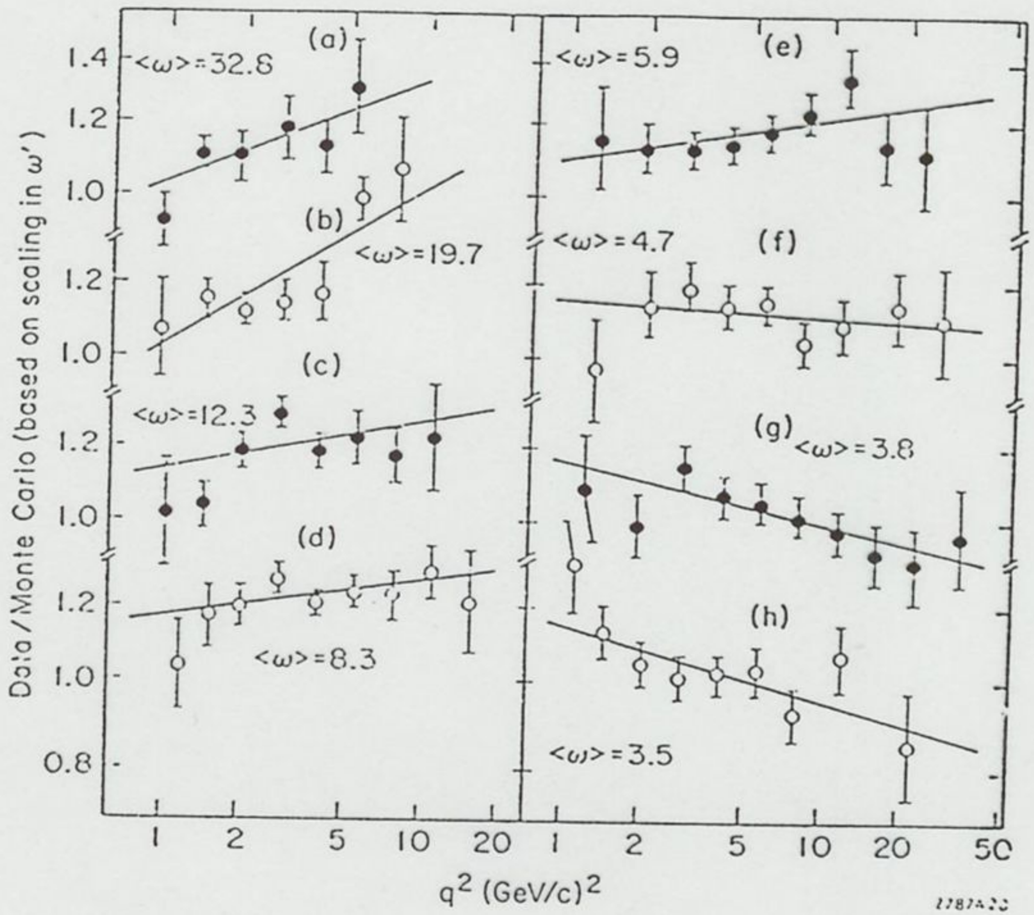
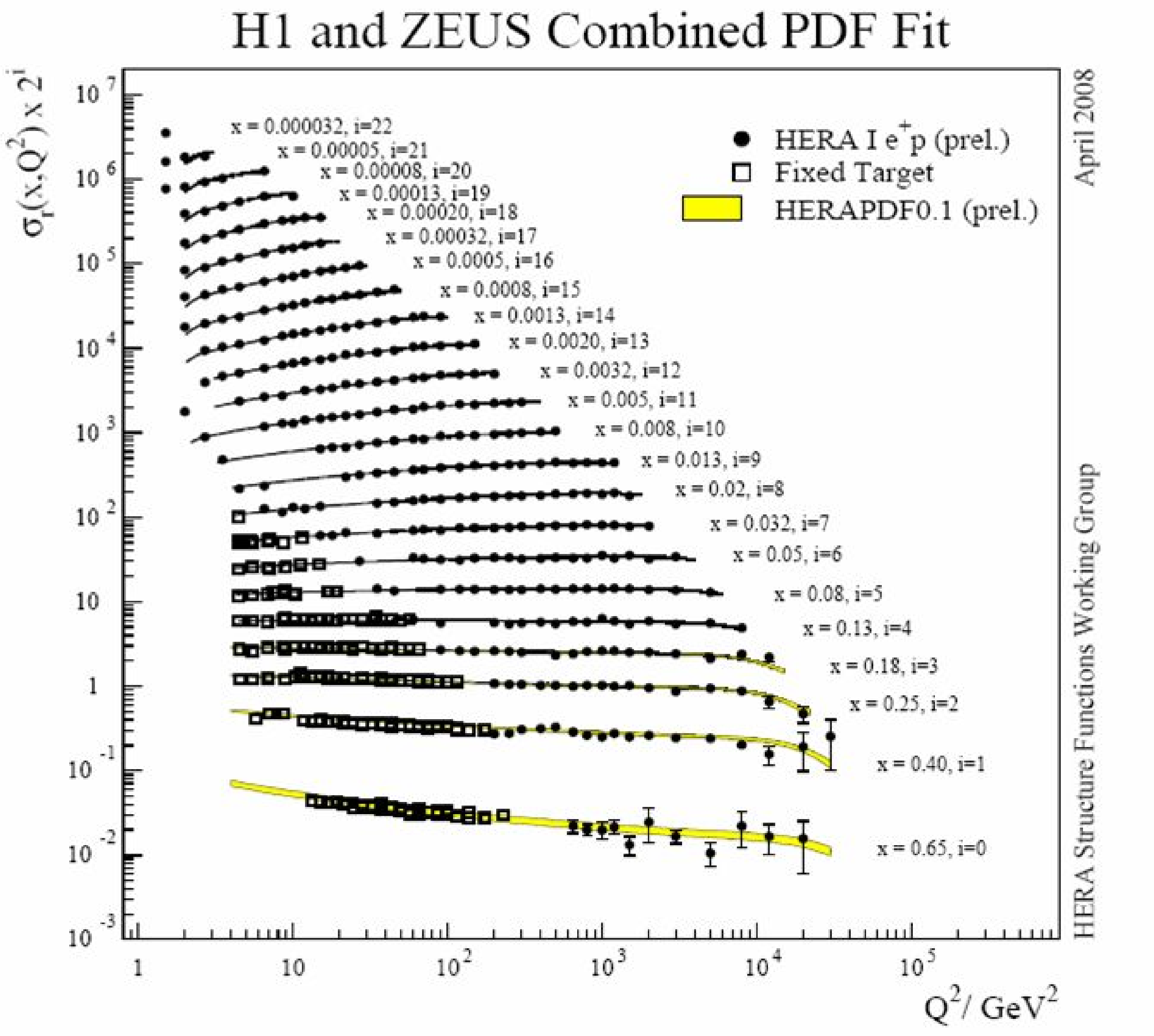
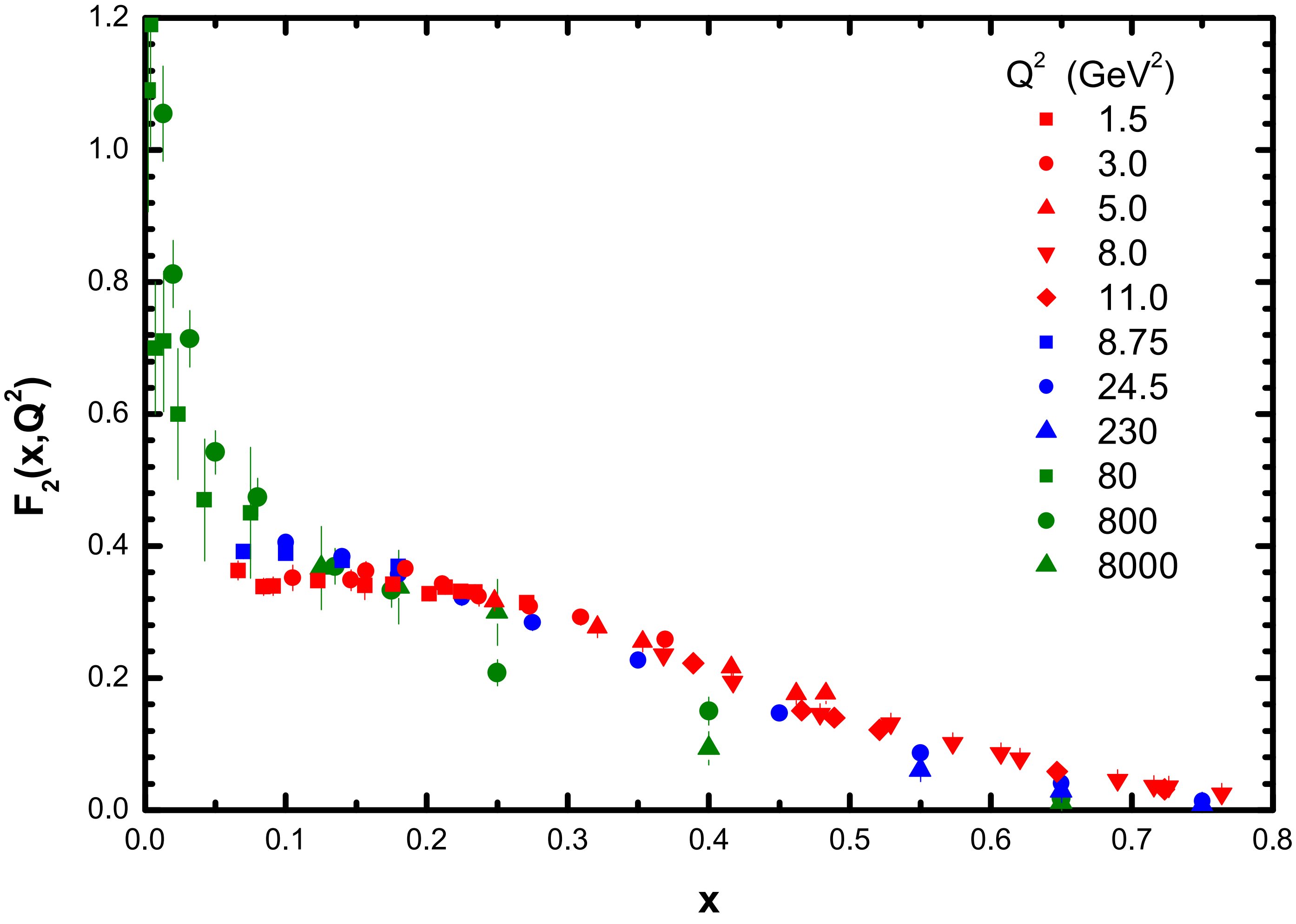


Fig. 7--Ratio of observed to "Monte Carloed" event rate vs. q^2 for eight ranges of ω.

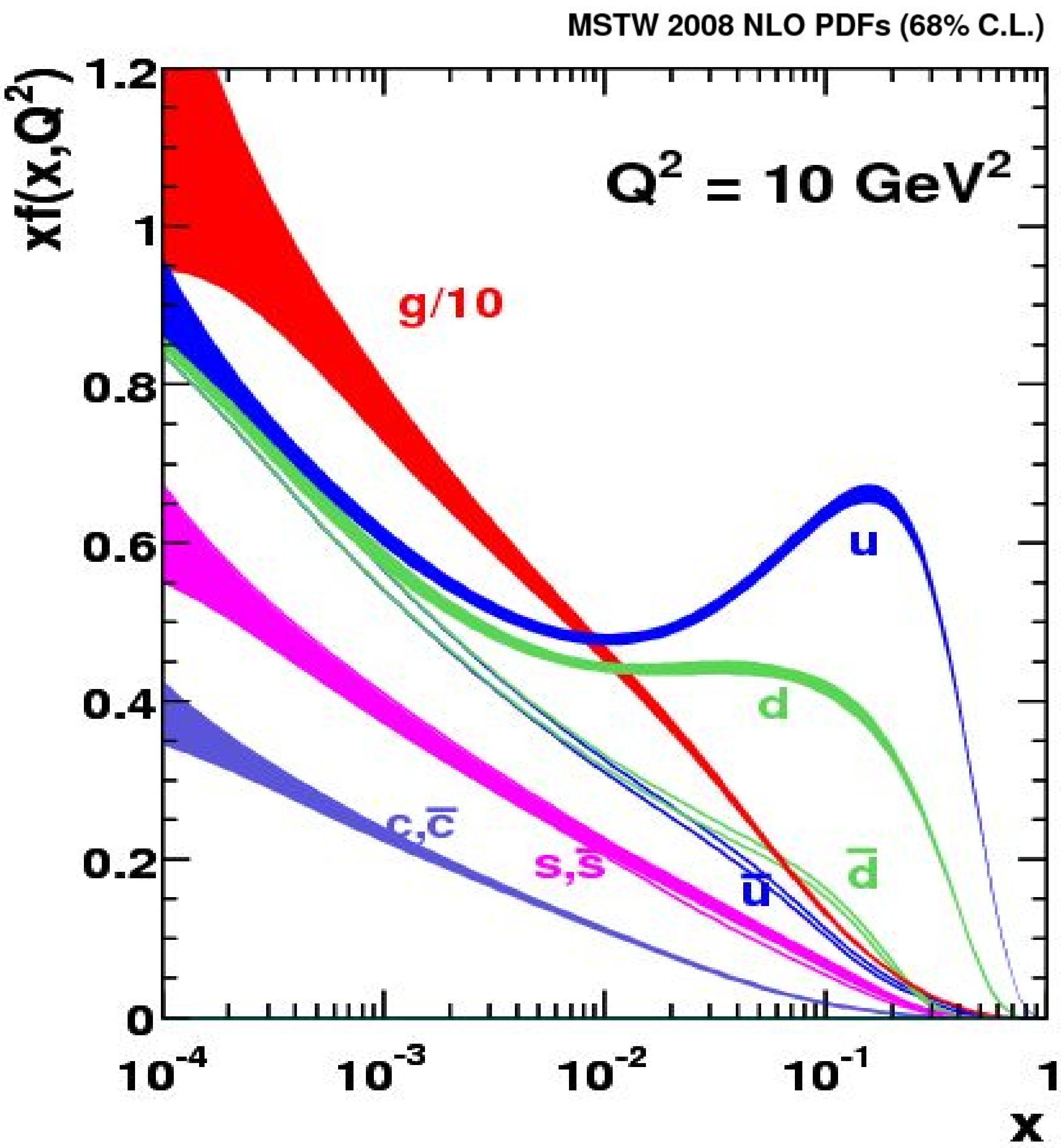
Scaling violations measured at HERA



40 years of Deep Inelastic Scattering



the data have errors, and therefore so do the pdfs…..



Conclusions

- Sum rules now used as constraints on parton distributions, or ways to measure α_s

- Historically sum rules provided

  - first suggestion of point-like behaviour (before the data)

  - direct evidence for quarks (spin, charges, baryon number)

  - first evidence for gluons