Path Group and quantum equivalence principle

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Motivation

- **New ideas** needed for Quantum Gravity
  - Physical ideas
  - Mathematical ideas
- One of the ideas - **no manifold !!!**
  - **Loop gravity**
  - Concept of **locality** **(cf. Giddings)**
- **Path Group** provides **geometry** (including **topology**) **without manifold**

Interpretation of derivatives

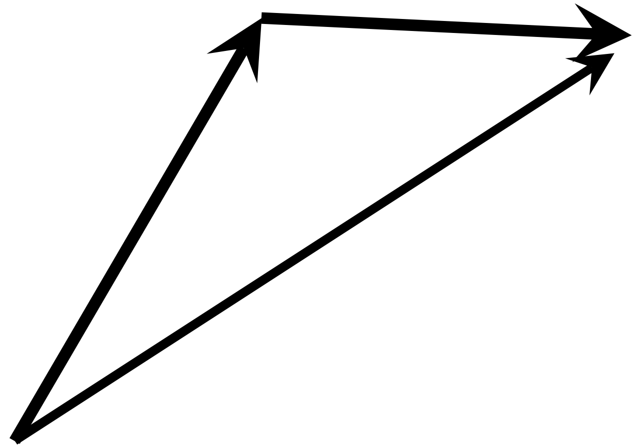
- **Derivatives**: generate translations

$$e^{\Delta x^\mu \, \partial_\mu} f(x) = f(x + \Delta x)$$

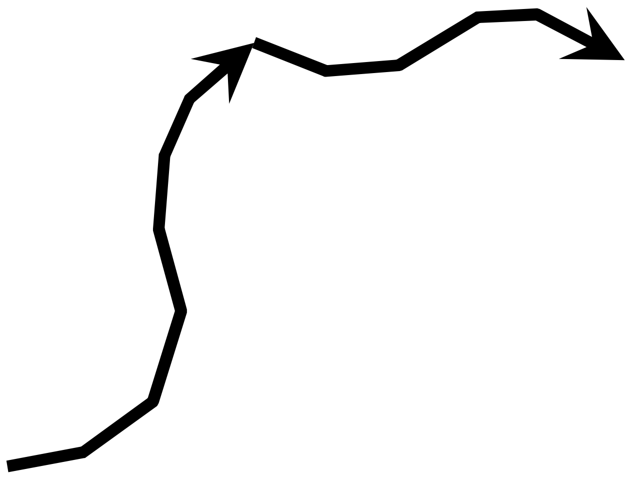
- **Covariant derivatives**: generate the action of paths
  - –gauge th: $U(p) = \mathcal{P} \exp \left\{ - \int_p d\xi^\mu \nabla_\mu \right\}$
  - –Gravity: $U(p) = \mathcal{P} \exp \left(\int_p d\xi^\alpha \, B_\alpha \right)$

Paths generalize translations

- Translations are vectors



- Paths are (classes of) curves

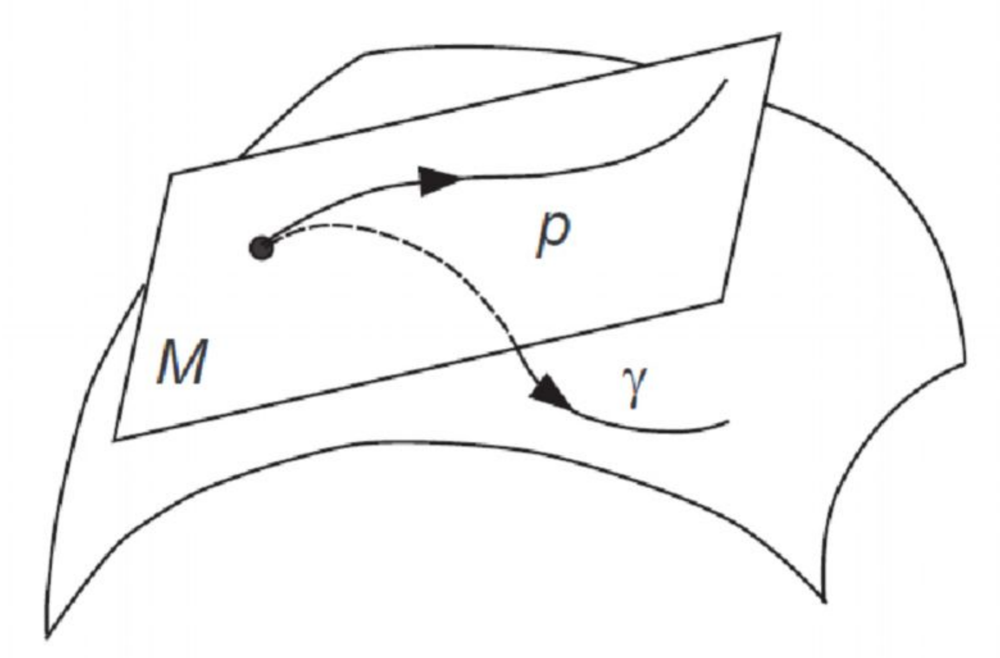


PG in gravity

- "Flat model" of a curve
- PG on fiber bundle (local frames)
- Generalized Poincare Group (PG+Lorentz group)
- Holonomy Subgroup in GPG

“Flat model” of a curve

- Mapping of a curve in the tangent space onto the curve in the curved space

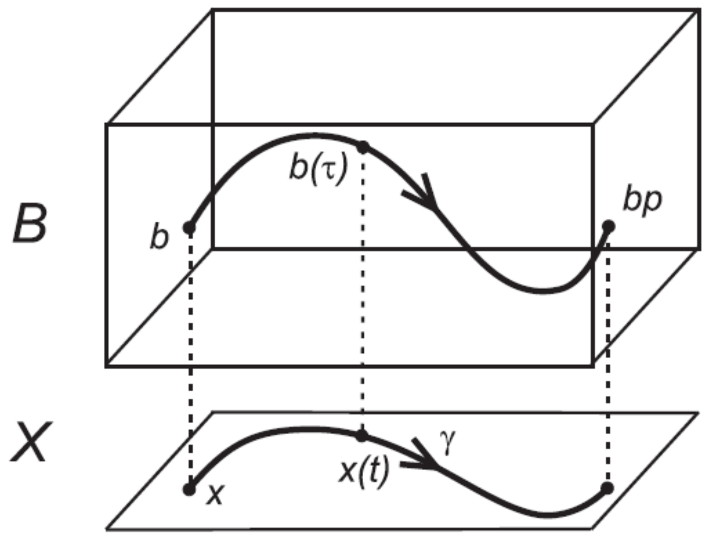


$$\dot{x}^{\mu}(\tau) = \dot{\xi}^{\alpha}(\tau) b^{\mu}_{\alpha}(\tau)$$

$$\dot{b}^{\lambda}_{\beta}(\tau) = \Gamma_{\mu\nu}(x(\tau))\, \dot{x}^{\mu}(\tau) b^{\nu}_{\beta}(\tau)$$

Action in the fiber bundle

- Fiber bundle of **local frames**
- Horizontal basis vector fields
- Ordered exponential along the path
- Path ***p*** is a flat model of ***γ***



$$\boldsymbol{U(p) = \mathcal{P}\exp\left(\int_p d\xi^\alpha\, B_\alpha\right)}$$

$$\boldsymbol{B_\alpha = b^\mu_\alpha\left(\frac{\partial}{\partial x^\mu} - \Gamma^\lambda_{\mu\nu}(x)\, b^\nu_\beta \frac{\partial}{\partial b^\lambda_\beta}\right)}$$

Generalized Poincare Group

- **Poincare Group**: semi-direct product of
  Translation Group by Lorentz group
- **GPG**: semi-direct product of
  Path Group by the Lorentz group

$$\boxed{Q = \Lambda \ltimes P}$$

- Group structure of the group Q:

$$q = p\lambda, \qquad \lambda\{\xi\}\lambda^{-1} = \{\lambda\xi\}$$

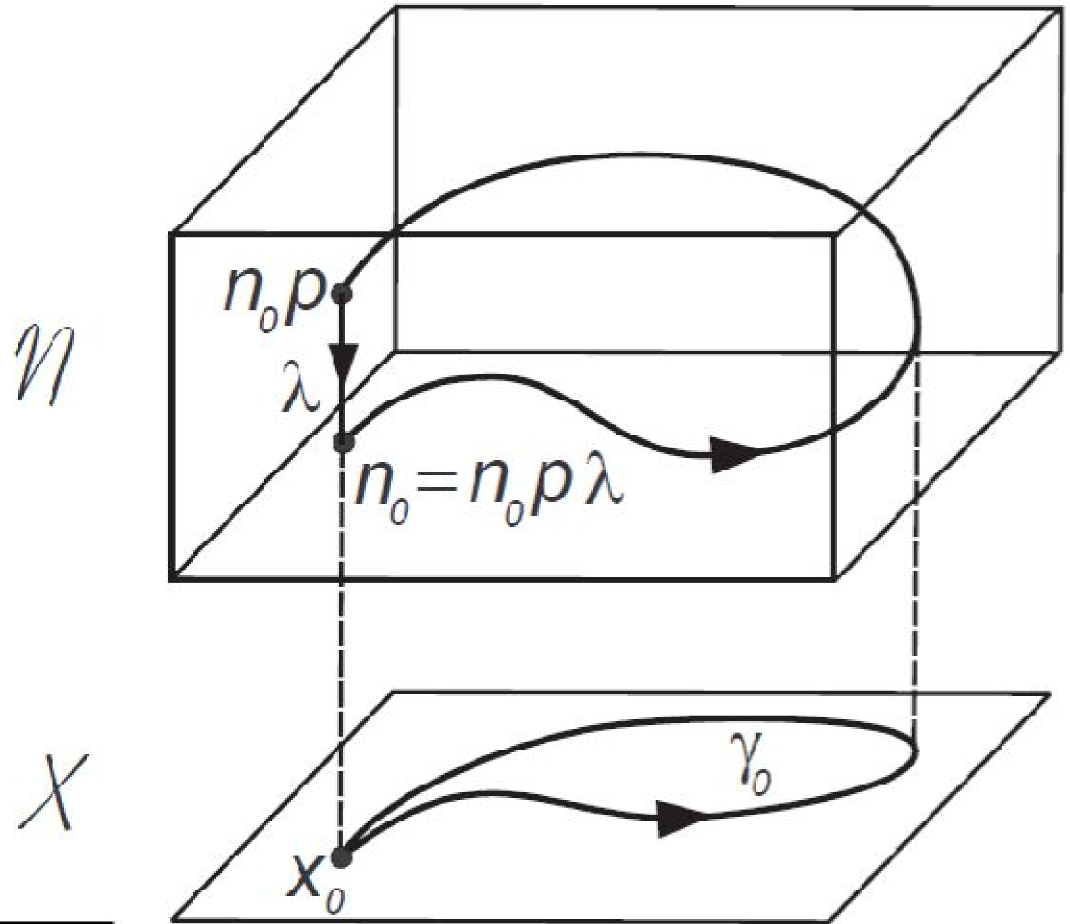
Holonomy subgroup *H*

- Stationary subgroup of a local basis n_0

$$\boxed{H \subset Q}$$

$$h = p\lambda \in H$$

if $$\boxed{n_0 \, p\lambda = n_0}$$



Quantum equivalence principle

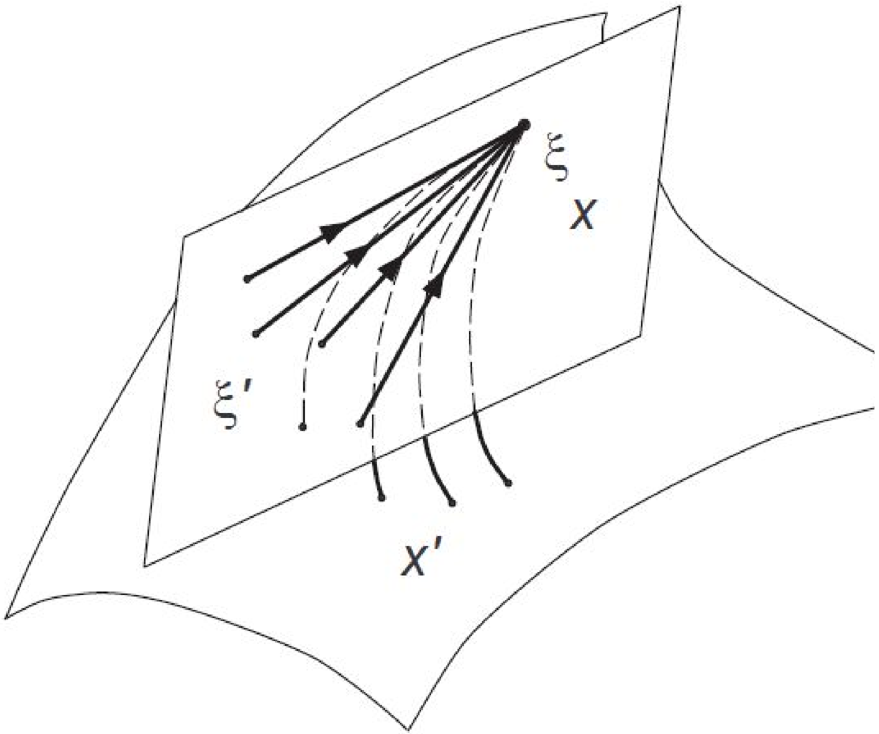
- Use flat models of trajectories
- Classical particle: straight line
- Quantum particle:
  Feynman path integral

MM 1973

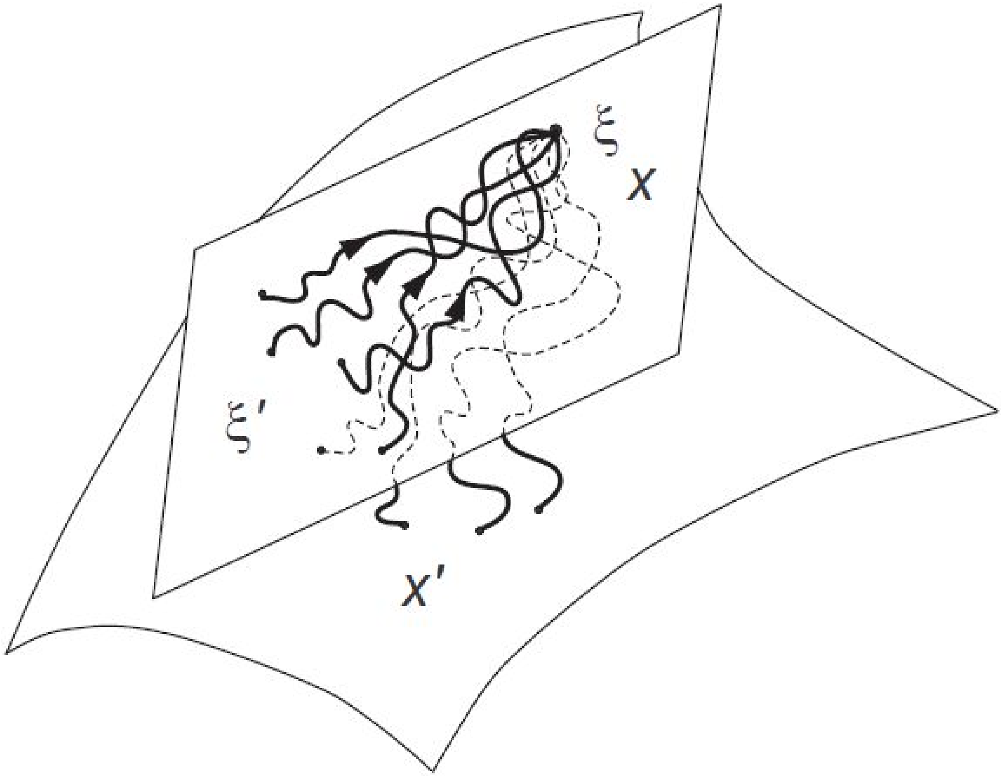
Pazma & Presnaider 1988, Kleinert 1995

Classical and quantum EP

- **Classical**: motion along **straight lines**



- **Quantum**: Feynman **path integral**



Path integral in a curved space

Topology in terms of GPG

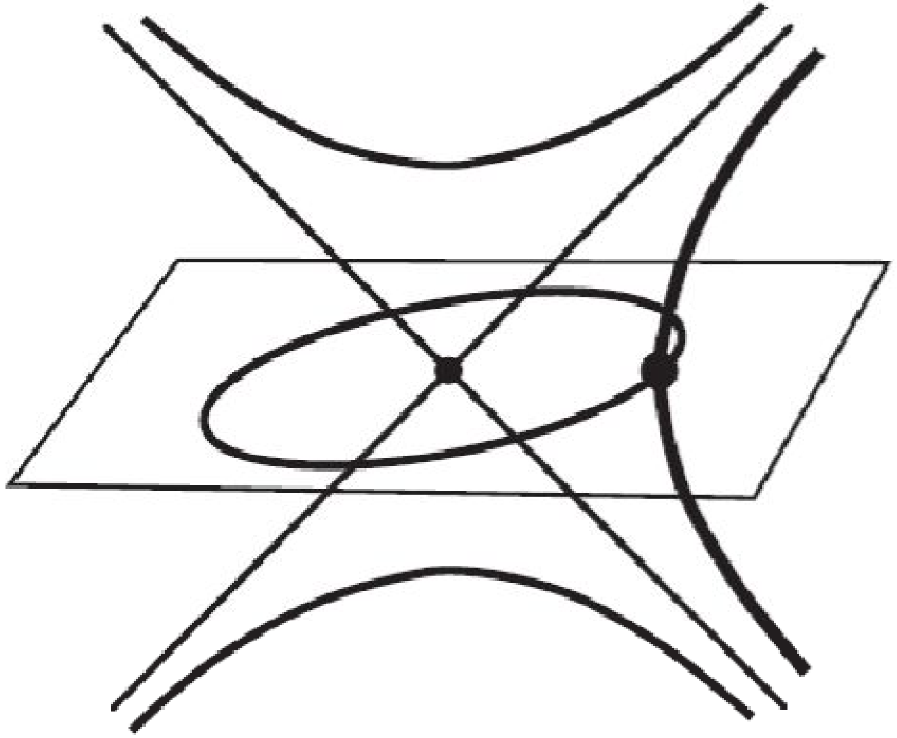
- PG: geometry including topology (holonomy subroup of GPG)
- Applications:
  - “Lorentz cone” and topological model of the Universe expansion
  - Gravitational thermal effects

Gravitational thermal effects

- Mathematics
  - Imaginary time (Euclidean loops)
  - Restricted path integral
  - Universal Rindler scheme
- Physics
  - Real or virtual thermal bath?
  - Is «second space» real?

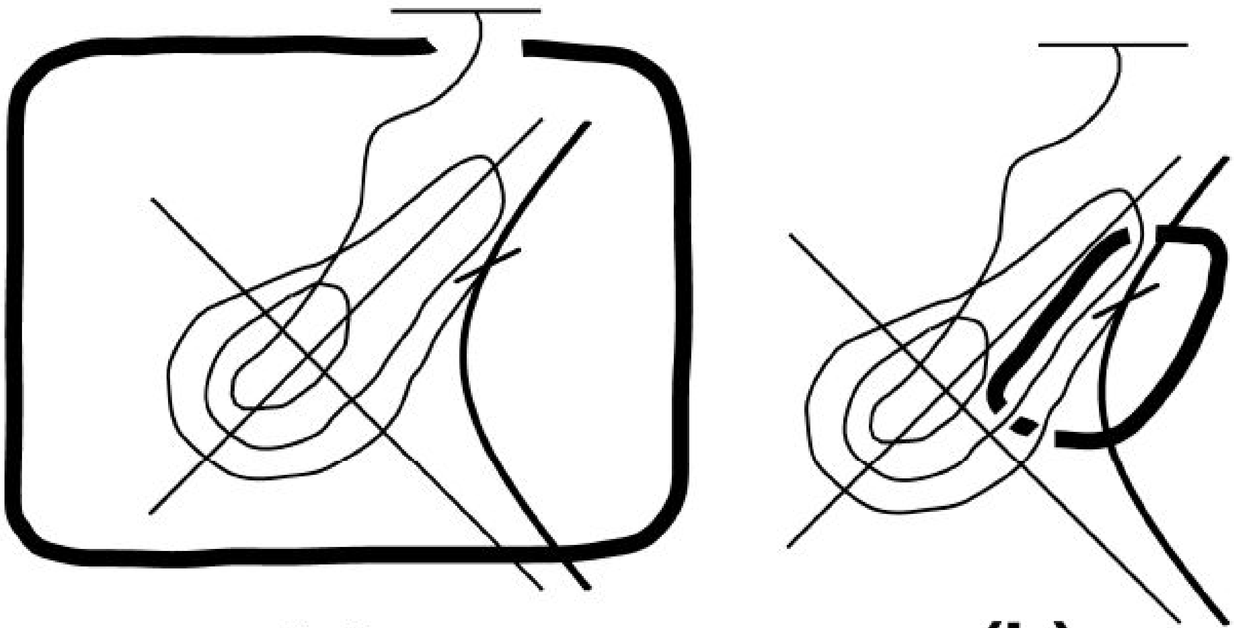
Observer far from BH

- **Detector far from BH** is described by the **corridor of paths**
- The detector may go into **Euclidean section** and pass along a **loop**
- $\Rightarrow$ Observation of thermal radiation



r = const

Accelerated observer (Unruh)

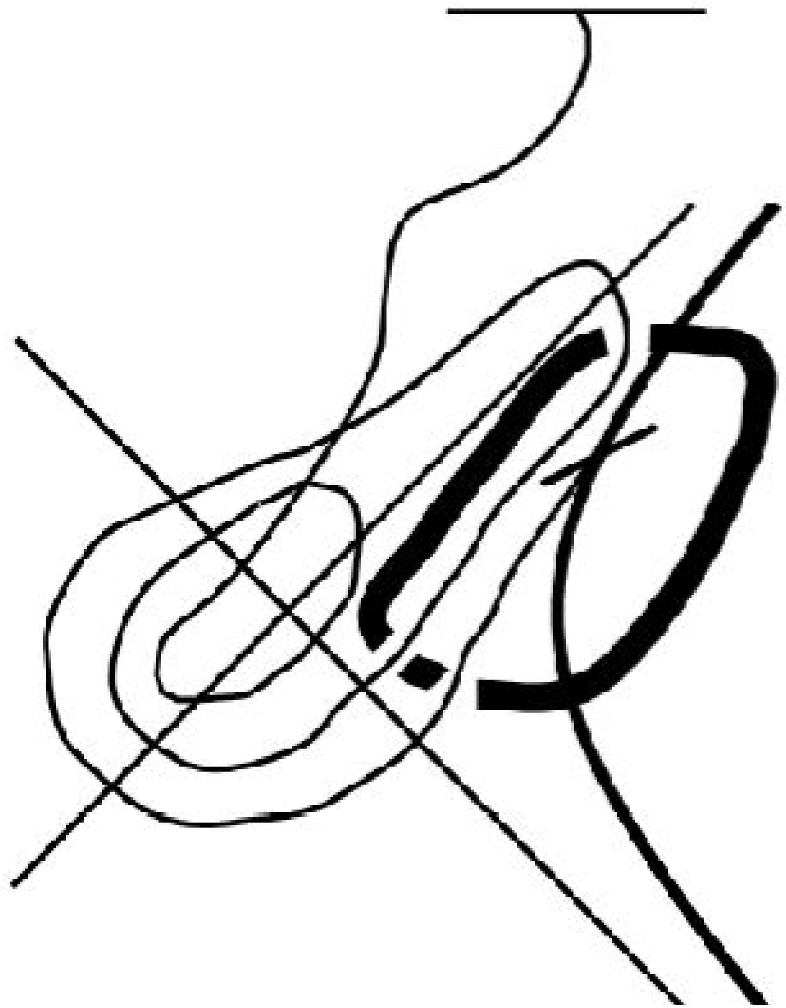


(a) (b)

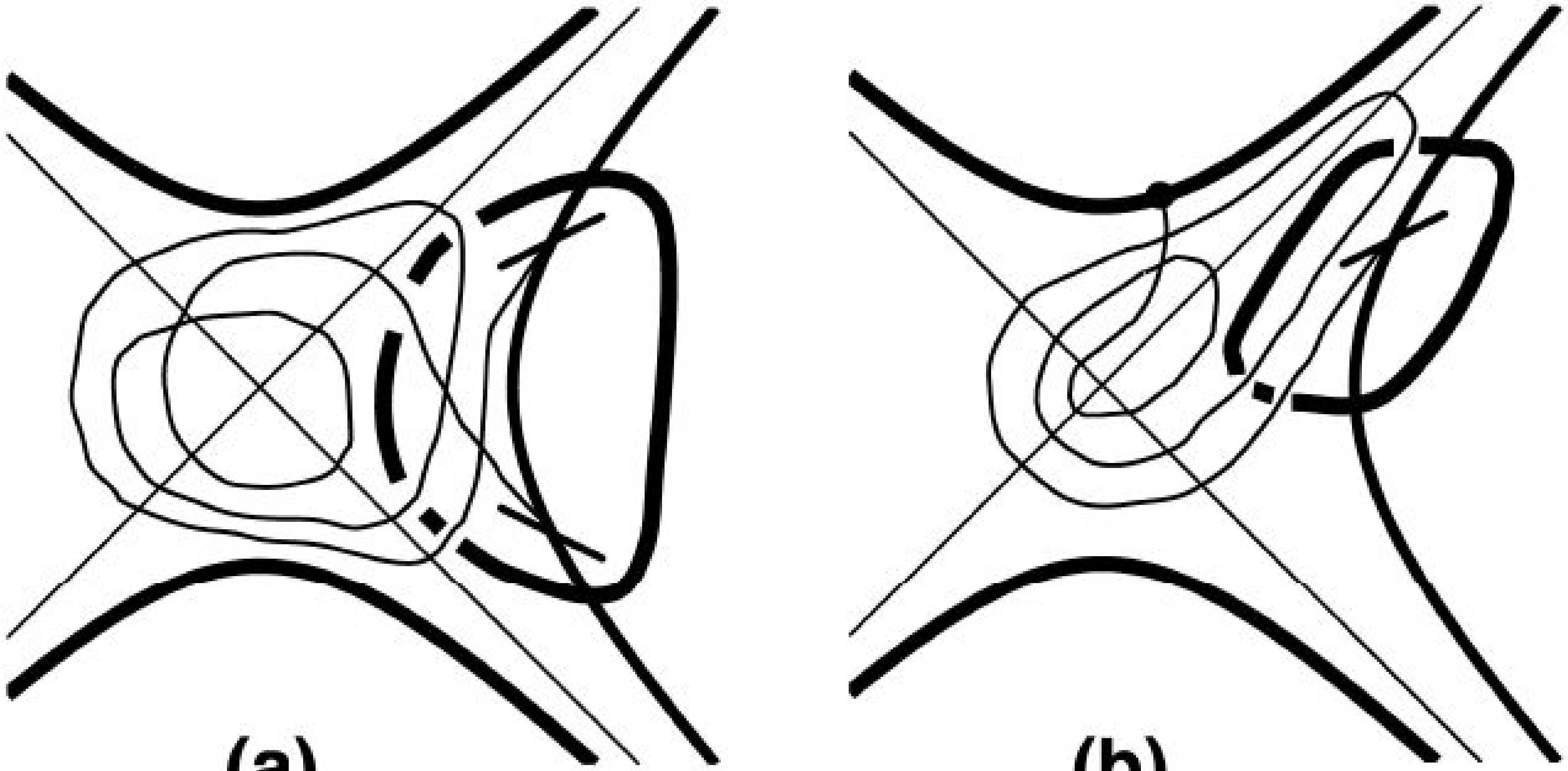
- (a) Wide corridor: processes of absorption and radiation of real particles (only statistics)
- (b) Narrow corridor: individual «thermal particles» are virtual

Is the Unruh energy real?

- Absorption of a particle by the accelerated observer
- The region restricting the paths is less than the wavelength
- The antiparticle is radiated
- Real energy is absorbed



Eternal Black Hole

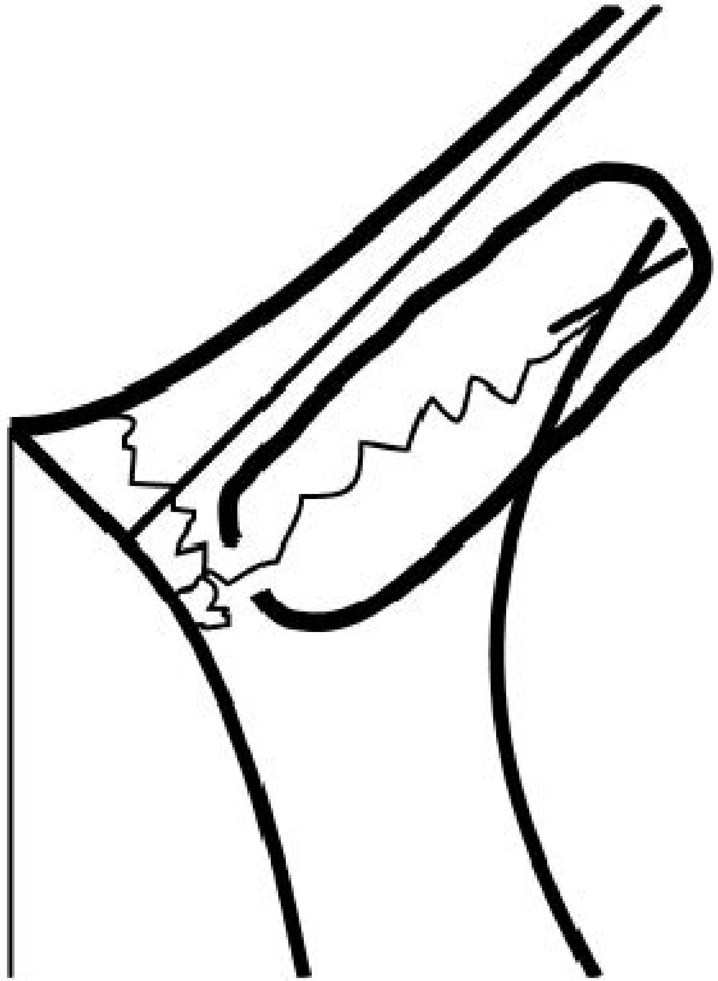


(a) (b)

Eternal BH is stable until its radiation is absorbed

Collapsing body

- Particle absorbed by the observer is real
- It may be traced back up to the point of its radiation
- BH created in the collapse is really evaporating



Conclusion

- PG and GPG: Minkowski structure in a curved space-time
- Gauge theory and gravity as representations of PG
- Holonomy subgroup in GPG: geometry and topology
- Applications for thermal effects

Literature

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