IV Sakharov Conference
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COMMENTS ON THERMODYNAMICS OF SUPERSYMMETRIC MATRIX MODELS

based on

A.S., Nucl. Phys. B818 (2009) 101 [arXiv:0812.4753[hep-th]].

STRING-FIELD DUALITY

allows to obtain strong results for certain SUSY field theories at strong coupling

- Most known : AdS/CFT

Circular Wilson loop

- exact result (all-order perturbative resummation)

$$\langle W\rangle_{\rm circle} \;=\; \frac{2I_1(\sqrt{\lambda})}{\sqrt{\lambda}} \qquad\qquad (\lambda = g^2 N_c).$$

- on the string side

$$\langle W\rangle_{\rm circle} \;=\; \sqrt{\frac{2}{\pi}} e^{\sqrt{\lambda}} \frac{1}{\lambda^{3/4}} \left[1 - \frac{3}{8\sqrt{\lambda}} + \ldots\right] \;.$$

- The leading term is obtained from the analysis of the $AdS_5 \times S^5$ solution of $10D$ supergravity.
- string corrections.

• cusp anomalous dimension and many other wonders.

One of them: Thermal internal energy

$$E = \frac{\pi^2 N^2}{2} T^4 \left[\frac{3}{4} + \frac{45}{32} \frac{\zeta(3)}{\lambda^{3/2}} + \ldots \right]$$

(the coefficient in front of T^4 is the coefficient in the Stefan-Boltzmann law).

• Derived via certain black brane solutions in 10D supergravity

- duality works also for other theories

1) Low-dimensional sisters of $N=4$ SYM.

2) $3D \;\; N=6$ theories

The youngest (prettiest ?) sister:

$10D$ SQM model

$$H \;=\; \frac{1}{2}E_i^aE_i^a + \frac{g^2}{4}f^{abe}f^{cde}A_i^aA_j^bA_i^cA_j^d + \frac{ig}{2}f^{abc}\lambda_\alpha^a(\Gamma_i)_{\alpha\beta}\lambda_\beta^bA_i^c \;,$$

where $i,j \;=\; 1,\ldots,9,\;\; a \;=\; 1,\ldots,N^2-1$, and $\alpha,\beta=1,\ldots,16$. E_i^a are canonical momenta for the bosonic dynamic variables A_i^a; λ_α^a are Majorana fermion variables lying in the **16**- plets of $SO(9)$.

- Dimensionful coupling constant g^2 gives an intrinsic energy scale

$$E_{\rm char} \sim (g^2N)^{1/3} \equiv \lambda^{1/3} \;.$$

THERMODYNAMICS:

weak coupling at $T \gg E_{\rm char}$,

$$\langle E \rangle_T \propto \#_{\rm d.o.f.} T$$

strong coupling at $T \ll E_{\rm char}$,

$$\left\langle \frac{E}{N^2} \right\rangle_{T \ll \lambda^{1/3}} \approx 7.41 \lambda^{1/3} \left(\frac{T}{\lambda^{1/3}} \right)^{14/5} \left[1 + O\left(\frac{T}{\lambda^{1/3}} \right)^{9/5} \right] .$$

Sketch of derivation

(Klebanov, Tseytlin, Kabat, Lifschytz, Hanada, Hyakutake, Nishimura, Takeuchi)

- black hole solution

$$
\begin{aligned}
ds^2_{\rm BH} &= \frac{r^{7/2}}{R^{7/2}}\left[-dt^2\left(1-\frac{r_0^7}{r^7}\right)\right]+\frac{R^{7/2}}{r^{7/2}}\left[\frac{dr^2}{1-\frac{r_0^7}{r^7}}+r^2 d\Omega_8^2\right] \\
e^{-2\phi(r)} &\propto (r/R)^{21/2} \qquad \text{(dilaton)}
\end{aligned}
$$

BH enthropy = horizon volume

$$
S_{BH}=V_{\rm horizon} = e^{-2\phi(r_0)}\sqrt{-g}(r_0) \propto r_0^{9/2}
$$

$T_{\rm Hawking}$ = grav. accel. at horizon

$$
T_{\rm Hawking} \propto a_{\rm horizon} \sim \left.\frac{dg_{00}}{dr}\right|_{r=r_0} \propto r_0^{5/2} \; .
$$

Getting rid of r_0, we obtain

$S \propto T^{9/5}$ and $\langle E \rangle_T \propto T^{14/5}$

verified numerically on the QM side

(Anagnostopoulos et al, 2007)

analytical understanding ?

A) Pure YM QM

- dicrete spectrum with $E_{\text{char}} \sim \lambda^{1/3}$

- $T \gg \lambda^{1/3} \longrightarrow \langle E \rangle_T \sim (3/4) N^2 (D{-}2) T \quad (D = 4, 6, 10)$

- $T \ll \lambda^{1/3} \longrightarrow \langle E \rangle_T \sim e^{-\lambda^{1/3}/T}$

B) SYM QM with $D = 4, 6$

- Discrete spectrum like in pure YM theory.
- Vacuum valleys ($[A_i, A_j] = 0$) and continuous spectrum
- Infinite contribution to the partition function:

$$Z = \int \frac{dpdx}{2\pi} E^{-\beta p^2/2} \; = \; L \sqrt{\frac{T}{2\pi}}$$

• In our case, $(D-1)(N-1)$ degrees of freedom and

$$Z_{\rm cont} \sim \left(\frac{T}{\mu}\right)^{(D-1)(N-1)/2}$$

(μ - infrared cutoff).

• When $N \to \infty$ and μ fixed, it is suppressed compared to

$$Z_{\rm discr.,\ highT} \sim \left(\frac{T}{\lambda^{1/3}}\right)^{\frac{3}{4}N^2(D-2)} ,$$

• The same pattern for

$$\langle E\rangle_T = -\frac{\partial}{\partial\beta}\ln Z$$

as for pure YM.

C) SYM QM with $D = 10$

- New feature: Normalized zero energy states
- For large $\vec{A}$ along the valley,

$$\Psi(A_i, \lambda) \sim \frac{1}{|\vec{A}|^9}$$

- Characteristic size of Ψ estimated from H_{eff} on the valley (Okawa, Yoneya),

$$H_{\text{eff}}(N) = \sum_{n=1}^{N} |\vec{E}^n|^2 + \frac{15}{16} \sum_{n>m}^{N} \frac{|\vec{E}^n - \vec{E}^m|^4}{g^3 |\vec{A}^n - \vec{A}^m|^7} + \ldots,$$

$\hat{\vec{A}} = \text{diag}(\vec{A}^1, \ldots, \vec{A}^N)$ and $\hat{\vec{E}} = \text{diag}(\vec{E}^1, \ldots, \vec{E}^N)$.

- Comparison of two terms gives

$$A_{\text{char}}^2 \sim \frac{N^{2/9}}{g^{2/3}} \sim N^{5/9} \lambda^{-1/3} .$$

• At $T > N^{-5/9}\lambda^{1/3}$, this family gives the contribution

$$Z(T) \;\sim\; \exp\left\{ N^2 \left(\frac{T}{\lambda^{1/3}}\right)^{9/5} - N \right\} \;,$$

which REPRODUCES

$S \propto T^{9/5}$ and $\langle E \rangle_T \propto T^{14/5}$

• exponential fall-off of $\langle E \rangle_T$ at $T < N^{-5/9}\lambda^{1/3}$.

TWO PROBLEMS

1) No numerical evidence for the growth of $A^2_{\rm char}$ with N.

2) String corrections to the strong coupling estimate $\langle E \rangle_T \propto T^{14/5}$ become important at $T \sim \lambda^{1/3} N^{-10/21}$ rather than $T \sim \lambda^{1/3} N^{-5/9}$.

$$\frac{10}{21} \neq \frac{5}{9}$$

$$\ldots$$