Formation of a pulsar magnetosphere from a strongly magnetized vacuum

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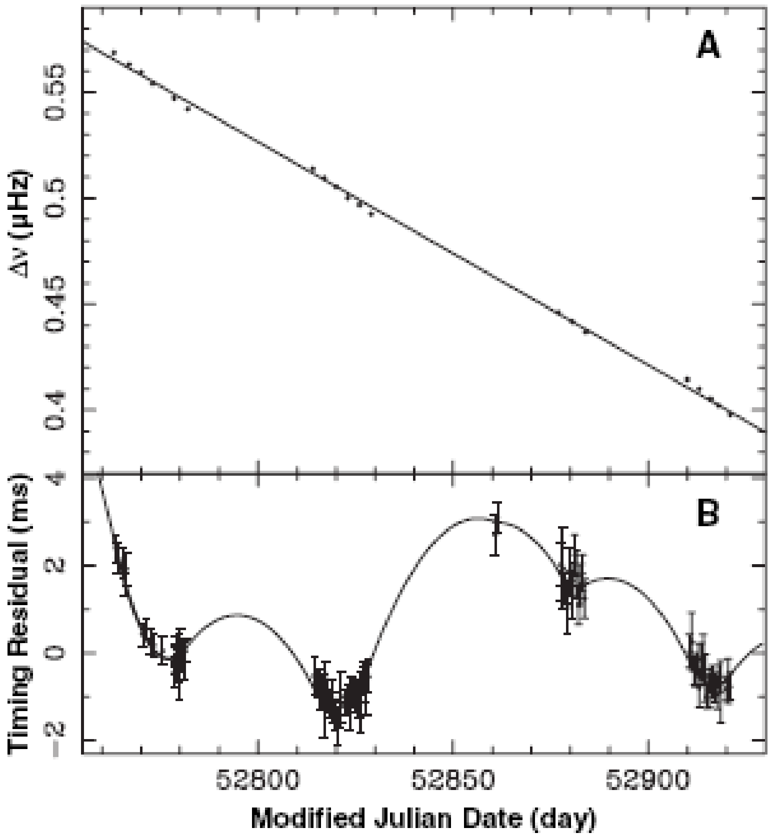
MIPT

On/off pulsars: B1931+24

Kramer et al., 2006



Active phases are 5 to 10 days long

Switching off is in less than 10 s

Undetectable for 25 to 35 days

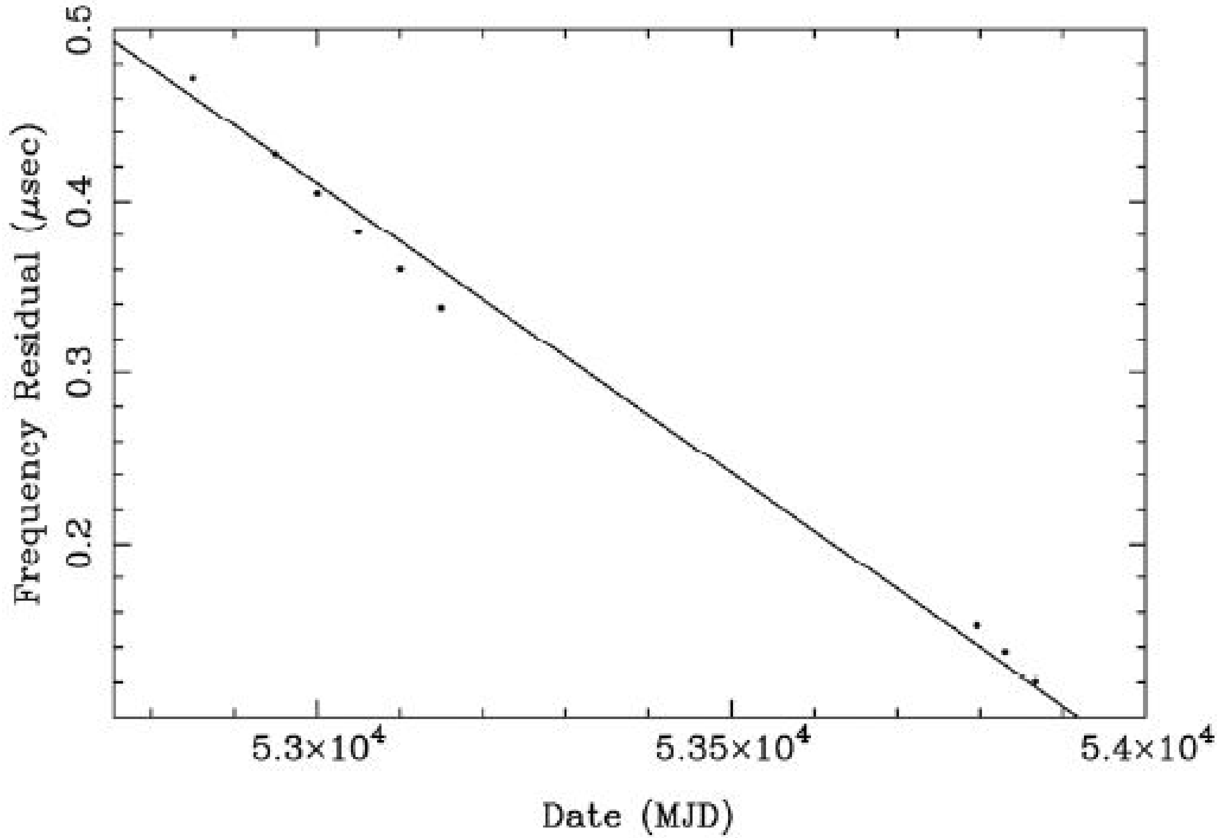
On-phase — rotation slows down 50% faster

On/off pulsars: B1931+24

Parameter	Value
Right ascension (J2000)	$19^h33^m37^s.832(14)$
Declination (J2000)	$+24°36'39''.6(4)$
Epoch of frequency (modified Julian day)	50629.0
Rotational frequency ν (Hz)	1.2289688061(1)
Rotational frequency derivative $\dot{\nu}$ (Hz s^{-1})	$-12.2488(10) \times 10^{-15}$
Rotational frequency derivative on $\dot{\nu}_{on}$ (Hz s^{-1})	$-16.3(4) \times 10^{-15}$
Rotational frequency derivative off $\dot{\nu}_{off}$ (Hz s^{-1})	$-10.8(2) \times 10^{-15}$
Dispersion measure DM (cm^{-3} pc)	106.03(6)
Flux density during on phases at 1390 MHz (μJy)	1000(300)
Flux density during off phases at 1390 MHz (μJy)	≤2
Flux density during on phases at 430 MHz (μJy)	7500(1500)
Flux density during off phases at 430 MHz (μJy)	≤40
Active duty cycle (%)	19(5)
Characteristic age τ (million years)	1.6
Surface magnetic field strength B (T)	2.6×10^8
Spin-down luminosity $\dot{E}$ (W)	5.9×10^{25}
Distance (kpc)	~4.6

On/off pulsars: J1832+0029

Lyne, 2006



On-state > 300 days

Off-state ~ 700 days

Increase in slow-down rate is similar to B1931+24

Nulling pulsars

Wang et al., 2007

(1) J2000 name	(2) B1950 name	(3) P (s)	(4) $\dot{P}$ (10^{-15})	(5) Age (10^6 yr)	(6) DM (cm^{-3} pc)	(7) S_{1400} (mJy)	(8) Int. (s)	(9) NF (%)	(10) Null len. (s)	(11) Null cycle (s)
J0846–3533	B0844–35	1.116	1.60	11.0	94	2.7	10	0	...	...
J1032–5911	B1030–58	0.464	3.00	2.4	418	0.93	20	0	...	...
J1049–5833	...	2.202	4.41	7.9	447	0.72	10	47 (3)	84 (86)	179 (127)
J1326–6700	B1322–66	0.543	5.31	1.6	210	11.0	0	9.1 (3)	1 (1)	14 (32)
J1401–6357	B1358–63	0.842	16.70	0.8	98	6.2	0	1.6 (1)	2 (3)	120 (95)
J1412–6145	...	0.315	98.70	0.1	515	0.47	20	0	...	...
J1502–5653	...	0.535	1.83	4.6	194	0.39	10	93 (4)	450 (335)	515 (360)
J1525–5417	...	1.011	16.20	1.0	235	0.18	20	16 (5)	28 (25)	138 (102)
J1648–4458	...	0.629	1.85	5.4	925	0.55	60	1.4 (11)	...	...
J1658–4306	...	1.166	42.80	0.4	845	0.80	60	0	...	...
J1701–3726	B1658–37	2.454	11.10	3.5	303	2.9	10	14 (2)	16 (9)	118 (91)
J1702–4428	...	2.123	3.30	10.2	395	0.38	30	26 (3)	42 (24)	149 (83)
J1703–4851	...	1.396	5.08	4.3	150	1.10	10	1.1 (4)	...	...
J1717–4054	B1713–40	0.887	...	...	307	54	10	>95	>7000	>7000
J1722–3632	B1718–36	0.399	4.46	1.4	416	1.60	20	0	...	...
J1727–2739	...	1.293	1.10	18.6	147	1.60	10	52 (3)	48 (37)	91 (58)
J1812–1718	B1809–173	1.205	19.10	1.0	254	1.00	20	5.8 (4)	23 (6)	359 (303)
J1820–0509	...	0.337	0.93	5.7	104	*	10	67 (3)	71 (69)	104 (68)
J1831–1223	...	2.857	5.47	8.3	342	1.2	20	4 (1)	23 (7)	448 (348)
J1833–1055	...	0.633	0.53	19.1	543	0.5	30	7 (2)	...	...
J1843–0211	...	2.027	14.40	2.2	442	0.93	30	6 (2)	46 (23)	826 (872)
J1916+1023	...	1.618	0.68	37.7	330	0.36	60	47 (4)	70 (43)	152 (53)
J1920+1040	...	2.215	6.48	5.4	304	0.57	10	50 (4)	43 (25)	85 (35)

RRATs

McLaughlin et al., 2006

11 sources characterized by single dispersed bursts

Durations are between 2 and 30 s

Average time intervals between bursts range from 4 min to 3 hours

Radio emission is detectable for <1 s per day

Periodicities in the range of 0.4-7 s

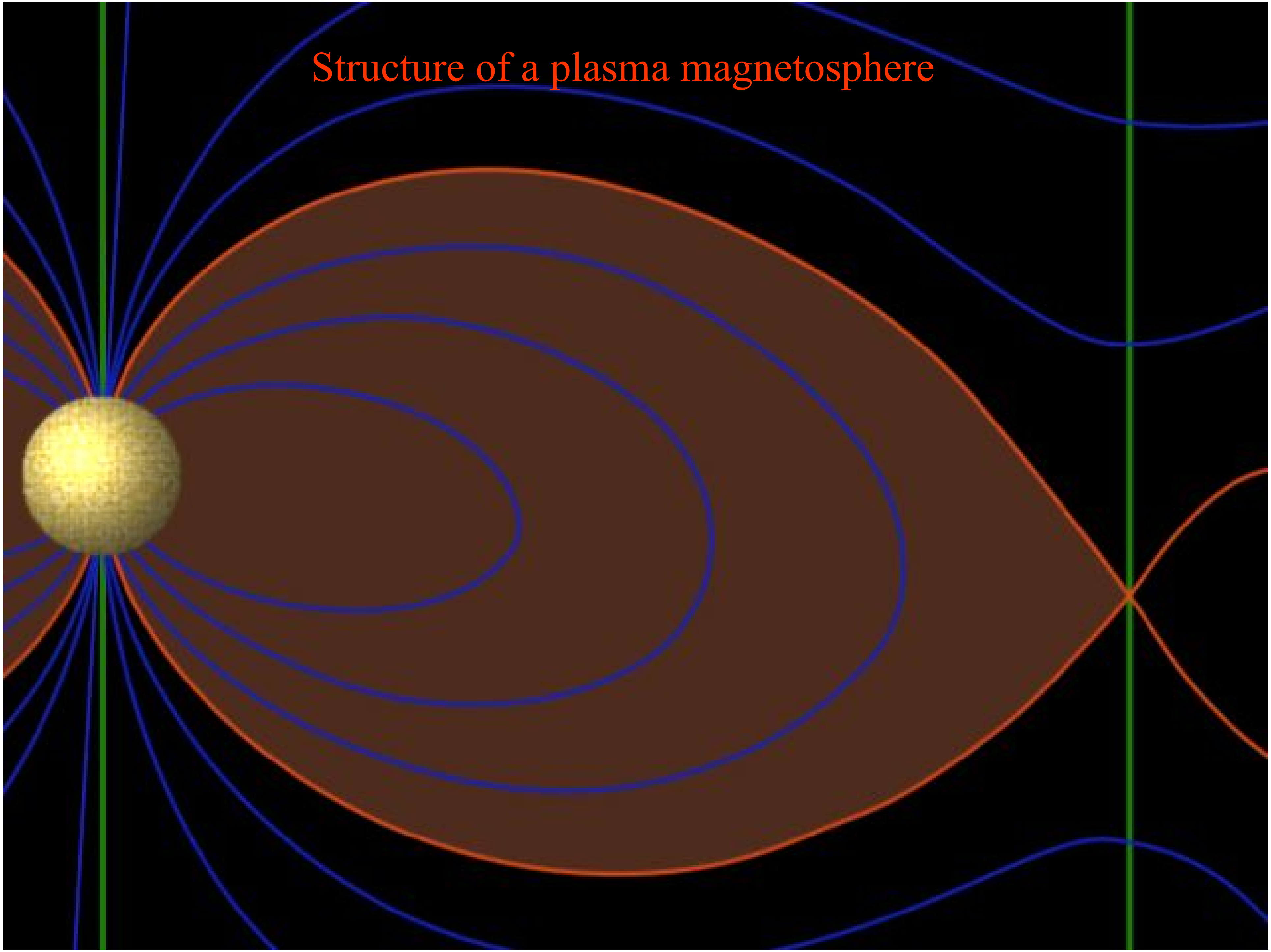
Pdot is measured for 3 sources

The inferred magnetic field reaches the magnetar values

RRATs

Name	RA (J2000) h m s	Dec (J2000) ° ′ ″	l °	b °	DM pc cm^{-3}	D kpc	w_{50} ms	S_{1400} mJy	N_p/T_{obs} hr^{-1}	N_{det}/N_{obs}
J0848–43	08:48(1)	–43:16(7)	263.4	0.2	293(19)	5.5	30	100	27/19	9/28
J1317–5759	13:17:46.31(7)	–57:59:30.2(6)	306.4	4.7	145.4(3)	3.2	10	1100	108/24	23/24
J1443–60	14:43(1)	–60:32(7)	316.2	–0.6	369(8)	5.5	20	280	32/41	17/25
J1754–30	17:54(1)	–30:11(7)	359.9	–2.2	98(6)	2.2	16	160	18/30	10/20
J1819–1458	18:19:33.0(5)	–14:58:16(32)	16.0	0.1	196(3)	3.6	3	3600	229/13	24/24
J1826–14	18:26(1)	–14:27(7)	17.2	–1.0	159(1)	3.3	2	600	18/17	8/12
J1839–01	18:39(1)	–01:36(7)	30.1	2.0	307(10)	6.5	15	100	8/13	1/10
J1846–02	18:46(1)	–02:56(7)	29.7	–0.1	239(10)	5.2	16	250	11/10	5/9
J1848–12	18:48(1)	–12:47(7)	21.1	–5.0	88(2)	2.4	2	450	10/8	5/9
J1911+00	19:11(1)	+00:37(7)	35.7	–4.1	100(3)	3.3	5	250	4/13	4/11
J1913+1333	19:13:17.69(6)	+13:33:20.1(7)	47.5	1.4	175.8(3)	5.7	2	650	66/14	7/10

Name	P s	w_{50}/P %	Epoch MJD	$\dot{P}$ 10^{-15} s s^{-1}	B 10^{12} G	τ_c Myr	$\dot{E}$ 10^{31} erg s^{-1}
J0848–43	5.97748(2)	0.50	53492	–	–	–	–
J1317–5759	2.6421979742(3)	0.38	53346	12.6(7)	5.83(2)	3.33(2)	2.69(1)
J1443–60	4.758565(5)	0.42	53410	–	–	–	–
J1754–30	0.422617(4)	3.79	53189	–	–	–	–
J1819–1458	4.263159894(6)	0.07	53265	576(1)	50.16(6)	0.1172(3)	24.94(5)
J1826–14	0.7706187(3)	0.26	53587	–	–	–	–
J1839–01	0.93190(1)	1.61	51038	–	–	–	–
J1846–02	4.476739(3)	0.36	53492	–	–	–	–
J1848–12	6.7953(5)	0.03	53158	–	–	–	–
J1913+1333	0.9233885242(1)	0.22	53264	7.87(2)	2.727(4)	1.860(6)	39.4(1)


Structure of a plasma magnetosphere

Structure of a vacuum magnetosphere

Deutsch, 1955

Magnetic fields

$$B_r = \frac{2m}{r^3}\left[\cos\theta\cos\theta_m + \sin\theta\sin\theta_m\cos(\varphi-\varphi_m)\right],$$

$$B_\theta = \frac{m}{r^3}\left[\sin\theta\cos\theta_m - \cos\theta\sin\theta_m\cos(\varphi-\varphi_m)\right],$$

$$B_\varphi = \frac{m}{r^3}\sin\theta_m\sin(\varphi-\varphi_m)$$

Electric fields

$$E_r = -k\frac{mR^2}{r^4}\left[\left(\frac{3}{2}\cos 2\theta + \frac{1}{2}\right)\cos\theta_m + \frac{3}{2}\sin 2\theta\sin\theta_m\cos(\varphi-\varphi_m)\right],$$

$$E_\theta = -k\frac{mR^2}{r^4}\left[\sin 2\theta\cos\theta_m + \left(\frac{r^2}{R^2} - \cos 2\theta\right)\sin\theta_m\cos(\varphi-\varphi_m)\right],$$

$$E_\varphi = k\frac{mR^2}{r^4}\left(\frac{r^2}{R^2} - 1\right)\cos\theta\sin\theta_m\sin(\varphi-\varphi_m).$$

The applicability criterion $(\Omega r_\perp/c)^2 \ll 1$

Structure of a vacuum magnetosphere

Particles are accumulated in the regions
with zero longitudinal accelerating electric fields

$$\mathbf{E}\cdot\mathbf{B} = -kr\frac{m^2}{r^6}\left[\left(1-\frac{R^2}{r^2}\right)\cos\theta''\sin\theta_m + \frac{R^2}{r^2}4\cos^2\theta'\cos\theta\right]$$

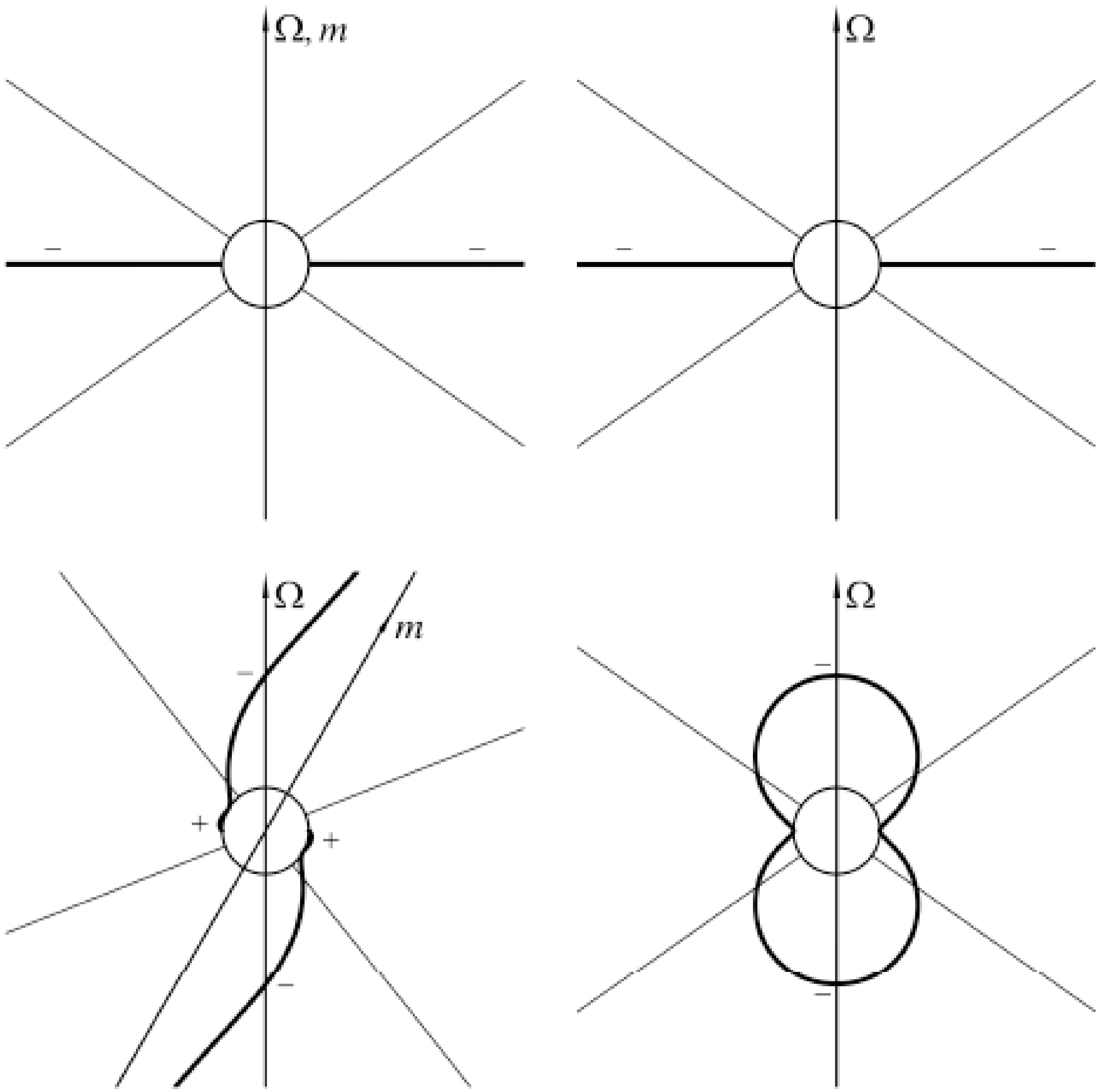
$$\cos\theta' = \cos\theta\cos\theta_m + \sin\theta\sin\theta_m\cos(\varphi-\varphi_m),$$

$$\cos\theta'' = -\cos\theta\sin\theta_m + \sin\theta\cos\theta_m\cos(\varphi-\varphi_m).$$

$$\cos\theta' = \mathbf{e}_r\cdot\mathbf{e}_m \qquad \cos\theta'' = \mathbf{e}_r\cdot\mathbf{e}_n$$

Equation for the force-free surface (FFS) $\mathbf{E}\cdot\mathbf{B} = 0$

$$r_{ffs}^2 = R^2\left(1-4\frac{\cos\theta\cos^2\theta'}{\sin\theta_m\cos\theta''}\right)$$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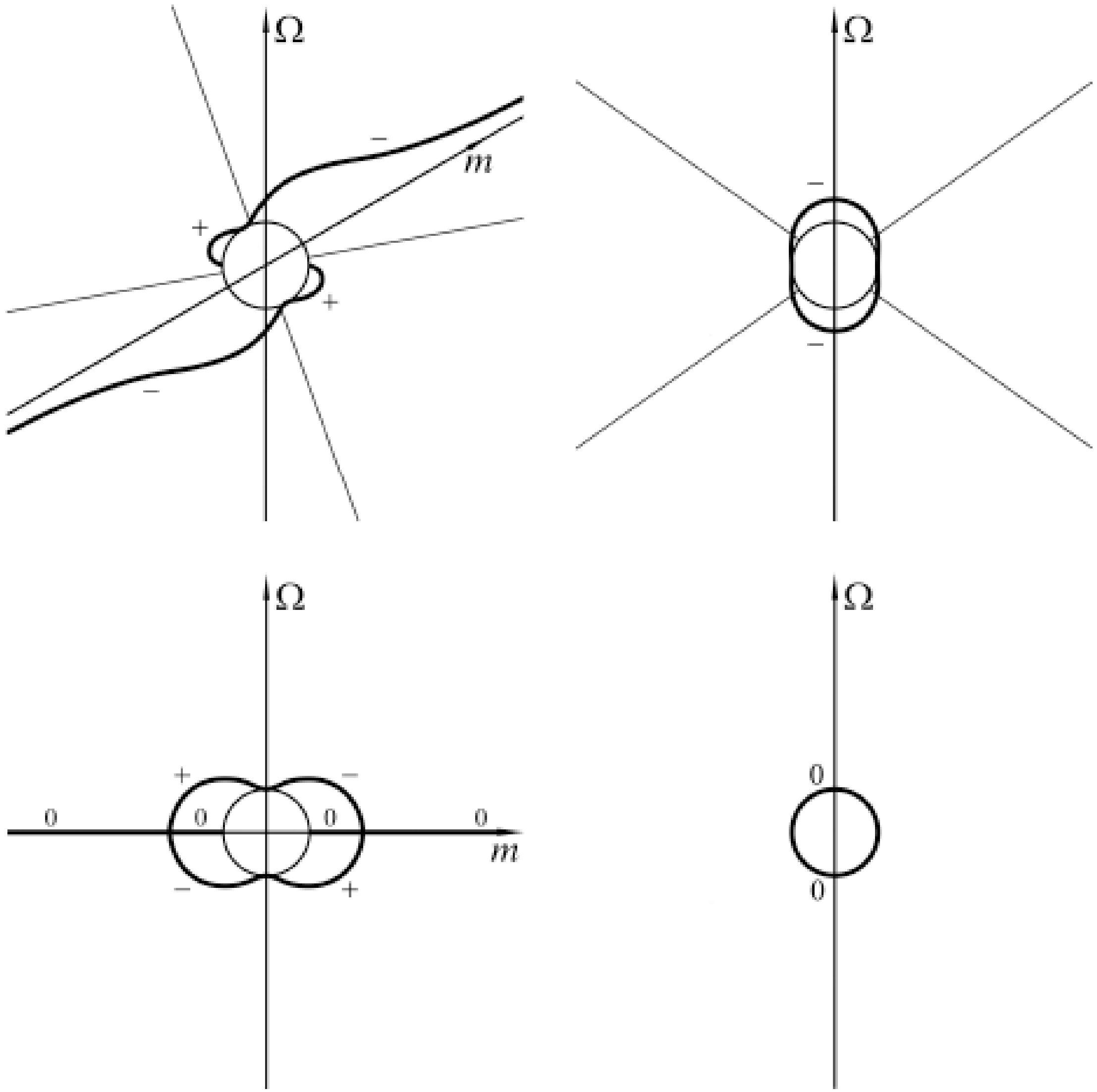

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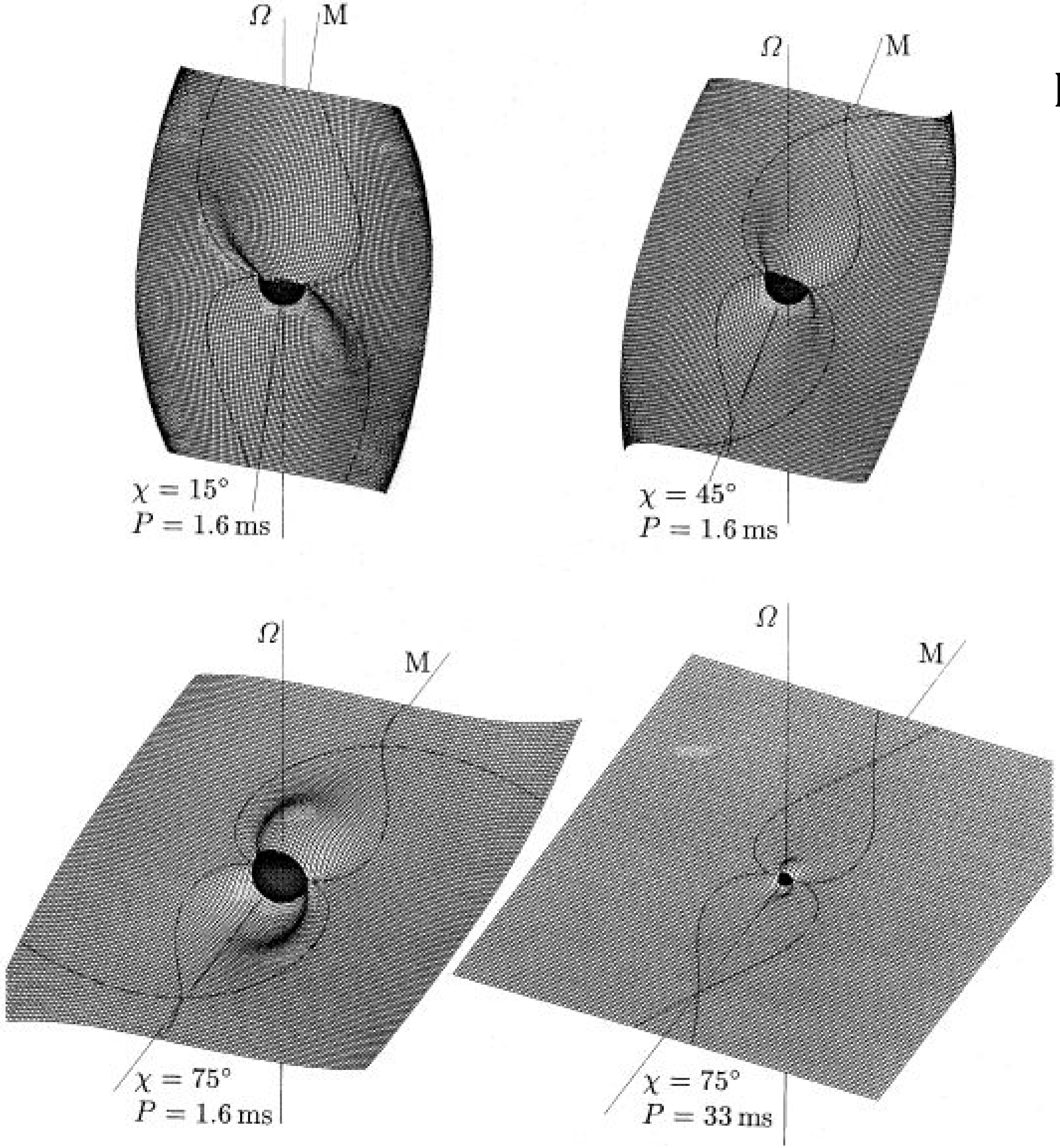
Finkbeiner et al., 1989



The Dirac-Lorentz equation

$$m_e \ddot{x}^i = \frac{2}{3c^3} e^2 \left[\dddot{x}^i + \frac{1}{c^2} \dot{x}^i \, \ddot{x}^k \ddot{x}_k \right] + F^i$$

4-vector

$$x^i = (ct, \mathbf{r}^T)^T$$

3D vector

$$\mathbf{r} = (x, y, z)^T$$

4-force

$$F^i = \frac{e}{c} F^{ik} \dot{x}_k$$

Electromagnetic tensor

$$F_{ik} = \partial A_k / \partial x^i - \partial A_i / \partial x^k$$

Electromagnetic 4-potential

$$A^i = (A^0, A^1, A^2, A^3)^T = (\phi, \mathbf{A}^T)^T$$

Electromagnetic fields

$$\mathbf{E} = -\nabla \phi - \frac{1}{c} \frac{\partial \mathbf{A}}{\partial t}, \qquad \mathbf{B} = \nabla \times \mathbf{A}.$$

The Dirac-Lorentz equation

$$\dot{x}^i = \gamma \frac{dx^i}{dt},$$
$$\ddot{x}^i = \gamma \frac{d\gamma}{dt}\frac{dx^i}{dt} + \gamma^2 \frac{d^2x^i}{dt^2},$$
$$\dddot{x}^i = \gamma \left(\frac{d\gamma}{dt}\right)^2 \frac{dx^i}{dt} + \gamma^2 \frac{d^2\gamma}{dt^2}\frac{dx^i}{dt} + 3\gamma^2 \frac{d\gamma}{dt}\frac{d^2x^i}{dt^2} + \gamma^3 \frac{d^3x^i}{dt^3},$$

$$\Downarrow$$

$$\frac{d\gamma}{dt} = \frac{2}{3}\alpha\gamma \left[\frac{d^2\gamma}{dt^2} - \gamma^3 \left(\frac{d\mathbf{v}}{dt}\right)^2\right] \pm \mathbf{v}\cdot\mathbf{E},$$
$$\gamma\frac{d\mathbf{v}}{dt} = \frac{2}{3}\alpha\gamma \left[3\frac{d\gamma}{dt}\frac{d\mathbf{v}}{dt} + \gamma\frac{d^2\mathbf{v}}{dt^2}\right] \pm \left[\mathbf{E} - \mathbf{v}(\mathbf{v}\cdot\mathbf{E}) + \mathbf{v}\times\mathbf{B}\right],$$

Motion of charged particles

Attainment of relativistic velocities

$$dp_{\|}/dt = E_{\|}.$$

$$\tau_{rel} \simeq \frac{1}{E_{\|}}$$

Surface fields

$$E_{surf} \simeq \frac{R}{R_c} B_{surf}$$

$$E_{surf} \sim 10^{-6} - 10^{-4} \quad \Rightarrow \quad \tau_{rel} \sim 10^{-17} - 10^{-15} \text{ s}$$

Accelerated particles produce curvature radiation

$$k_c = \frac{3}{2} \frac{\gamma^3}{\rho},$$

Motion of charged particles

Quasistationary motion

$$d\gamma/dt = d^2\gamma/dt^2 = 0$$

Power of accelerating electric field forces =
curvature radiation intensity

$$\frac{2}{3}\alpha\gamma_0^4\left(\frac{d\mathbf{v}}{dt}\right)^2 = \mathbf{v}\cdot\mathbf{E}.$$

Due to ultrarelativistic motion $d\mathbf{v}/dt = \mathbf{n}/\rho$

$$\gamma_0 = \left(\frac{3}{2\alpha}E_\parallel\rho^2\right)^{1/4}.$$

$$E_\parallel \sim 10^{-4} \quad \rho \sim R \simeq 2.6\times 10^{16} \quad \Rightarrow \quad \gamma_0 \sim 6\times 10^7$$

Motion of charged particles

$$d\gamma/dt = E_{\|}$$

Time of the complete acceleration

$$\tau_{st} \simeq \frac{\gamma_0}{E_{\|}}.$$

$$\tau_{st} \sim 10^{-9}\ \mathrm{s}$$

Energy adjustment

$$\frac{d^2\delta\gamma}{dt^2} - \frac{3}{2\alpha\gamma_0}\frac{d\delta\gamma}{dt} - \frac{4\gamma_0^2}{\rho^2}\delta\gamma = 0.$$

Roots of characteristic equation

$$\lambda_1 = \frac{3}{2\alpha\gamma_0}, \qquad \lambda_2 = -\frac{8}{3}\alpha\frac{\gamma_0^3}{\rho^2}.$$

The applicability criterion $\frac{8}{3}\alpha E_{\|} \ll 1$ is always satisfied.

Energy adjustment

Eliminating self-accelerating solution yields

$$\delta\gamma = \delta\gamma_i \, e^{-t/\tau_0}$$

Decay time

$$\tau_0 = \frac{3}{8\alpha}\frac{\rho^2}{\gamma_0^3}$$

$$\tau_0 \sim 10^{-10} - 10^{-7} \text{ s}$$

While the particles approach the FFS, the quasistationary condition is disrupted. The self-adjustment time increases and reaches the characteristic time of the electric field change.

Motion of charged particles

$$\gamma \frac{d\mathbf{v}}{dt} = \frac{2}{3}\alpha\gamma \left[3\frac{d\gamma}{dt}\frac{d\mathbf{v}}{dt} + \gamma\frac{d^2\mathbf{v}}{dt^2}\right] \pm \left[\mathbf{E} - \mathbf{v}(\mathbf{v}\cdot\mathbf{E}) + \mathbf{v}\times\mathbf{B}\right]$$

$\gamma_0/R \sim 10^{-9}$

$d\gamma/dt \sim \gamma_0/R$

$|d\mathbf{v}/dt| \sim 1/R$

$|d^2\mathbf{v}/dt^2| \sim 1/R^2$

$\gamma_0 \sim 10^8$

$R \sim 10^{17}$

$E \sim 10^{-4}$

!$\Downarrow$!

$$\alpha\frac{\gamma_0}{R} \ll 1$$

$\Downarrow$

$$\alpha\gamma_0^2/R^2$$

$\Downarrow$

$$\alpha\frac{\gamma_0^2}{R^2} \ll E,$$

Equation of motion

$$\gamma \frac{d\mathbf{v}}{dt} = \mathbf{E} - \mathbf{v}(\mathbf{v}\cdot\mathbf{E}) + \mathbf{v}\times\mathbf{B}.$$

Motion of charged particles

$$\mathbf{v}_\perp = \frac{1}{B^2}\left[\mathbf{E} - \mathbf{v}(\mathbf{v}\cdot\mathbf{E}) - \gamma\frac{d\mathbf{v}}{dt}\right] \times \mathbf{B}$$

Total velocity of a particle

$$\mathbf{v} = \mathbf{b} + \mathbf{v}_e + \mathbf{v}_c;$$

Electric drift

$$\mathbf{v}_e = \frac{\mathbf{E}\times\mathbf{B}}{B^2},$$

$$v_e \sim E/B$$

$$R/R_c \sim 10^{-4}$$

Centrifugal drift

$$\mathbf{v}_c = \frac{\gamma}{B}\,\mathbf{b}\times\frac{d\mathbf{b}}{dt}$$

$$\mathbf{v}_c = \frac{\gamma}{B\rho}\,\boldsymbol{\nu};$$

$$v_c \sim 10^{-8}\text{–}10^{-7}$$

$\boldsymbol{\nu} = \mathbf{b}\times\mathbf{n}$ — binormal vector

$\mathbf{v} = \mathbf{b} + \mathbf{v}_e$ — motion is virtually along magnetic field lines

Motion in rotating frame

Transformation of the velocity and acceleration

$$\mathbf{v} = \mathbf{v}' + \mathbf{v}_{tr},$$

$$\frac{d\mathbf{v}}{dt} = \frac{d\mathbf{v}'}{dt'} + 2\,\mathbf{\Omega} \times \mathbf{v}' + \mathbf{\Omega} \times \mathbf{v}_{tr},$$

$$\Downarrow$$

$$\gamma \frac{d\mathbf{v}'}{dt'} = \mathbf{E} + \mathbf{v}_{tr} \times \mathbf{B} - \mathbf{v}'(\mathbf{v}' \cdot \mathbf{E}) + \mathbf{v}' \times \mathbf{B}$$

We neglect the translation $\mathbf{\Omega} \times \mathbf{v}_{tr}$ and Coriolis $2\,\mathbf{\Omega} \times \mathbf{v}'$ accelerations with respect to the relative acceleration $d\mathbf{v}'/dt'$

$$|d\mathbf{v}'/dt'| \sim 1/R \qquad |2\,\mathbf{\Omega} \times \mathbf{v}'| \sim 1/R_c$$

Motion in rotating frame

$$\mathbf{v}' = d\mathbf{v}'/dt' = 0$$

Effective electric field

$$\mathbf{E}^{eff} = \mathbf{E} + \mathbf{v}_{tr} \times \mathbf{B}$$

Equilibrium condition $\mathbf{E}^{eff} = 0$

Equilibrium points

$$r = R, \quad \theta = \frac{\pi}{2} \qquad \text{(equator)},$$

$$r = R, \quad \theta' = \frac{\pi}{2} \qquad \text{(magnetic equator)},$$

$$\frac{r_+^2}{R^2} = \frac{3 + \cos\theta_m}{1 - \cos\theta_m} \qquad \text{(open surfaces)},$$

$$(\theta, \varphi) = \left\{ \left(\frac{\theta_m}{2}, \varphi_m \right), \left(\pi - \frac{\theta_m}{2}, \pi + \varphi_m \right) \right\},$$

$$\frac{r_-^2}{R^2} = \frac{3 - \cos\theta_m}{1 + \cos\theta_m} \qquad \text{(domes)},$$

$$(\theta, \varphi) = \left\{ \left(\frac{\pi}{2} + \frac{\theta_m}{2}, \varphi_m \right), \left(\frac{\pi}{2} - \frac{\theta_m}{2}, \pi + \varphi_m \right) \right\}.$$

Particle oscillation near the FFS

$$\mathbf{E} = \mathbf{E}_0 + (\mathbf{x}' \cdot \nabla)\mathbf{E}_0,$$

$$\mathbf{B} = \mathbf{B}_0 + (\mathbf{x}' \cdot \nabla)\mathbf{B}_0,$$

$$\mathbf{E}^{eff} = \mathbf{E}_0^{eff} + (\mathbf{x}' \cdot \nabla)\mathbf{E}_0^{eff},$$

We find the particular solution describing the motion with constant velocity

$$\mathbf{v}_0' = \mathrm{const}$$

$$\frac{d\mathbf{x}_0'}{dt'} = \mathbf{v}_0',$$

$$0 = \mathbf{E}_0^{eff} + (\mathbf{x}_0' \cdot \nabla)\mathbf{E}_0^{eff} + \mathbf{v}_0' \times \mathbf{B}_0 + \mathbf{v}_0' \times (\mathbf{x}_0' \cdot \nabla)\mathbf{B}_0.$$

$$\Downarrow$$

$$\mathbf{E}_0^{eff} + \mathbf{v}_0' \times \mathbf{B}_0 + t\Big[(\mathbf{v}_0' \cdot \nabla)\mathbf{E}_0^{eff} + \mathbf{v}_0' \times (\mathbf{v}_0' \cdot \nabla)\mathbf{B}_0\Big] = 0.$$

Particle oscillation near the FFS

Two conditions should be simultaneously satisfied

$$\mathbf{E}_0^{eff} + \mathbf{v}_0' \times \mathbf{B}_0 = 0,$$

$$(\mathbf{v}_0' \cdot \nabla)\mathbf{E}_0^{eff} + \mathbf{v}_0' \times (\mathbf{v}_0' \cdot \nabla)\mathbf{B}_0 = 0.$$

The first equation determines the transverse velocity component

$$\mathbf{v}_\perp' = \frac{\mathbf{E}_0^{eff} \times \mathbf{B}_0}{B_0^2},$$

However, the longitudinal component is still arbitrary:

$$\mathbf{v}_0' = v_{||}'\mathbf{b} + \mathbf{v}_\perp'$$

The second equation fixes the longitudinal component

$$v_{||}' = -\frac{\mathbf{v}_\perp' \cdot \nabla(\mathbf{E}_0 \cdot \mathbf{B}_0)}{\mathbf{b} \cdot \nabla(\mathbf{E}_0 \cdot \mathbf{B}_0)}.$$

The velocity vector lies on the FFS

$$\mathbf{v}_0' \cdot \nabla(\mathbf{E}_0 \cdot \mathbf{B}_0) = 0.$$

Particle oscillation near the FFS

The general solution $\mathbf{x}' = \mathbf{x}'_0 + \mathbf{x}'_1, \qquad \mathbf{v}' = \mathbf{v}'_0 + \mathbf{v}'_1,$

$$\gamma \frac{d\mathbf{v}'_1}{dt'} = (\mathbf{x}'_1 \cdot \nabla)\mathbf{E}_0^{eff} + \mathbf{v}'_0 \times (\mathbf{x}'_1 \cdot \nabla)\mathbf{B}_0 + \mathbf{v}'_1 \times \mathbf{B} - \mathbf{v}'_1(\mathbf{v}'_1 \cdot \mathbf{E}),$$

Non-relativistic case $(v'_1 \ll 1)$

$$\frac{d\mathbf{v}'_1}{dt'} = (\mathbf{x}'_1 \cdot \nabla)\mathbf{E}_0^{eff} + \mathbf{v}'_0 \times (\mathbf{x}'_1 \cdot \nabla)\mathbf{B}_0.$$

Non-relativistic oscillation frequency

$$\omega^2 = -\frac{\mathbf{b} \cdot \nabla(\mathbf{E}_0 \cdot \mathbf{B}_0)}{\mathbf{B}_0}.$$

$$\omega \sim \sqrt{\frac{B}{R_c}}. \qquad \nu \sim 1 - 10 \text{ GHz}$$

Particle oscillation near the FFS

Criterion for the applicability of nonrelativistic approximation

$$x_1' \ll l_{nro}$$

where

$$l_{nro} \simeq \frac{1}{\omega}.$$

$$l_{nro} \sim 1 \text{ cm}$$

General case

$$\gamma \frac{d\mathbf{v}_1'}{dt'} = (\mathbf{x}_1' \cdot \nabla)\mathbf{E}_0^{eff} + \mathbf{v}_0' \times (\mathbf{x}_1' \cdot \nabla)\mathbf{B}_0 - \mathbf{v}_1'(\mathbf{v}_1' \cdot \mathbf{E}).$$

$$\frac{dv_1'}{dt'} = -\frac{\omega^2}{\gamma^3} x_1'.$$

Ultrarelativistic approach is valid if $l_a \ll A.$ $\quad l_a \simeq \frac{l_{nro}^2}{A}.$

therefore

$$x_1' = \frac{2A}{\pi} \arcsin \sin\left(\frac{\pi t}{2A}\right)$$

Particle oscillation near the FFS

The equation of oscillations has the first integral

$$C = \gamma + \frac{\omega^2 (x_1')^2}{2}.$$

After change of variables

$$\sin\phi_0 = \frac{\kappa}{a}, \qquad \sin\phi = \frac{\omega}{\sqrt{2C}}\frac{x_1'}{a},$$

$$\kappa = \sqrt{\frac{C-1}{C+1}}, \qquad a = \sqrt{\frac{C-1}{C}}$$

we have

$$t = \frac{\sqrt{2C}}{\omega}\frac{R(\phi,\kappa)}{\sin\phi_0}.$$

$$R(\phi,\kappa) = E(\phi,\kappa) - \cos^2\phi_0 F(\phi,\kappa)$$

where the elliptic integrals are

$$F(\phi,\kappa) = \int_0^\phi \frac{d\alpha'}{\sqrt{1-\kappa^2\sin^2\alpha'}},$$

$$E(\phi,\kappa) = \int_0^\phi \sqrt{1-\kappa^2\sin^2\alpha'}\,d\alpha'.$$

Particle oscillation near the FFS

Oscillation period

$$T = 4\frac{\sqrt{2C}}{\omega}\frac{\mathrm{R}(\kappa)}{\sin\phi_0},$$

where

$$\mathrm{R}(\kappa) = \mathbf{E}(\kappa) - \cos^2\phi_0 \mathbf{K}(\kappa)$$

$$\mathbf{K}(\kappa) = F(\pi/2, \kappa) \text{ и } \mathbf{E}(\kappa) = E(\pi/2, \kappa)$$

In non-relativistic case

$$C \simeq 1, \kappa \simeq 0, \sin\phi_0 \simeq 1/\sqrt{2}$$

$$\mathbf{K}(0) = \mathbf{E}(0) = \pi/2$$

therefore

$$T = 2\pi/\omega$$

In ultrarelativistic case

$$C \simeq \omega^2 A^2/2,$$

$$\kappa \simeq \sin\phi_0 \simeq 1 \text{ и } \mathbf{E}(1) = 1$$

therefore

$$T = 4A$$

Particle oscillation near the FFS

The exact solution: $$x_1' = A\sin\phi,$$

$$\phi = Q\left(\frac{\omega t}{\sqrt{2(C+1)}}\right),$$

$Q(z)$ is inverse function for $R(\phi,\kappa)$

$z = R(Q(z),\kappa)$ for any $z \in \mathbb{R}$.

$$Q(\mathbb{R}) = \mathbb{R}.$$

Oscillatory character of motion:

$$Q(z + 2n\mathrm{R}(k)) = Q(z) + \pi n.$$

$$\phi(t+nT/2) = \phi(t)+\pi n$$

Asymptotics

$$Q(z) = \begin{cases} 2\,z, & \kappa = 0, \\ \arcsin(z - 2h(z)) + \pi h(z), & \kappa \to 1, \end{cases}$$

where $$h(z) = \left\lfloor \frac{z+1}{2} \right\rfloor$$

Energy losses due to oscillations

The equation for the energy evolution:

$$\frac{dC}{dt} = \frac{2}{3}\alpha\gamma\left[\frac{d^2\gamma}{dt^2} - \frac{\gamma^3}{\rho^2}\right]$$

The applicability criterion for adiabatic approach:

$$\frac{dC}{dt} \ll \frac{C}{T}.$$

Averaging over the oscillation period and using the ultrarelativistic solution yields

$$\langle\gamma^4\rangle = \frac{128}{315}C^4, \qquad \left\langle\gamma\frac{d^2\gamma}{dt^2}\right\rangle = \frac{2}{3}C\left(\frac{d^2C}{dt^2} - \omega^2\right)$$

$$\frac{dC}{dt} = -\frac{4}{9}\alpha\omega^2 C\left[1 + \frac{64}{105}\frac{C^3}{\omega^2\rho^2}\right]$$

Energy losses due to oscillations

Normalized first integral

$$\varepsilon(t) = \left[\left(1 + \varepsilon_0^{-3}\right) e^{3t/\tau_d} - 1\right]^{-1/3}$$

$$C_{curv} = \frac{105^{\,1/3}}{4}(\omega\rho)^{2/3} \approx 1.2\,(\omega\rho)^{2/3}$$

The damping time constant

$$\tau_d = \frac{9}{4}\frac{1}{\alpha\omega^2}.$$

$$\varepsilon(t) = \varepsilon_0 e^{-t/\tau_d} \qquad (C_0 \ll C_{curv}).$$

In this case the curvature radiation is small as compared to bremsstrahlung

Energy losses due to oscillations

$$\varepsilon(t) = \varepsilon_0 \left(1 - \frac{t}{\tau_p}\right) \qquad (C_0 \gg C_{curv},\ t \ll \tau_p),$$

where $\tau_p = \frac{\tau_d}{\varepsilon_0^3}$. is the time of switching to the power decay

$$\varepsilon(t) = \left(3\frac{t}{\tau_d}\right)^{-1/3} \qquad (C_0 \gg C_{curv},\ \tau_p \ll t \ll \tau_d).$$

In this case curvature emission dominates..
It is equal to bremsstrahlung when

$$\tau_{curv} = \frac{\ln 2}{3}\,\tau_d \approx 0.23\,\tau_d.$$

When t > τ_d , we have the exponential decay with the bremsstrahlung loss domination

$$\varepsilon(t) = e^{-t/\tau_d} \qquad (C_0 \gg C_{curv},\ t \gtrsim \tau_d).$$

The restriction on the oscillation amplitude from the adiabatic condition

$$A \ll A_{\max} = \left(\frac{945}{128\alpha}\right)^{1/7} \omega^{-6/7}\rho^{2/7} \approx 2.6\,\omega^{-6/7}\rho^{2/7}.$$

The oscillation period should be less than the characteristic time constant for energy decay

$$T \ll \tau_p,$$

The oscillation parameters

$$C_{curv} \sim 10^4$$

$$A_{curv} \sim 1 \text{ m}$$

$$C_{\max} \sim (3 \times 10^6) - 10^7$$

$$A_{\max} \sim 10 - 100 \text{ m}$$

$$\nu \sim 10 - 100 \text{ MHz}$$

$$\tau_{curv} \sim \tau_d \sim 10 - 1000 \text{ s}$$

$$\tau_p \sim 10^{-8} - 10^{-4} \text{ s}$$

Capturing charged particles

The quasistationary condition is violated when

$$\tau_0 = l_c.$$

Self-consistent mean free path:

$$l_c = \left(\frac{3}{512\alpha}\right)^{1/7} \omega^{-6/7}\rho^{2/7} \approx \omega^{-6/7}\rho^{2/7}$$

At this point, two conditions hold:
i) condition for quasistationary motion

$$\gamma_c = 4\,\omega^2 l_c^2$$

ii) condition for adiabatic oscillation

$$C = \frac{9\,\omega^2 l_c^2}{2}$$

The distance of the first-time particle deviation from the FFS:

$$A_c = 3\,l_c \approx 2.9\,\omega^{-6/7}\rho^{2/7}$$

Particle trajectories at the FFS

$$\mathbf{v}_0' \cdot \mathbf{E}^{eff} = 0.$$

Effective electric field is potential:

$$\mathbf{E}^{eff} = \nabla \xi,$$

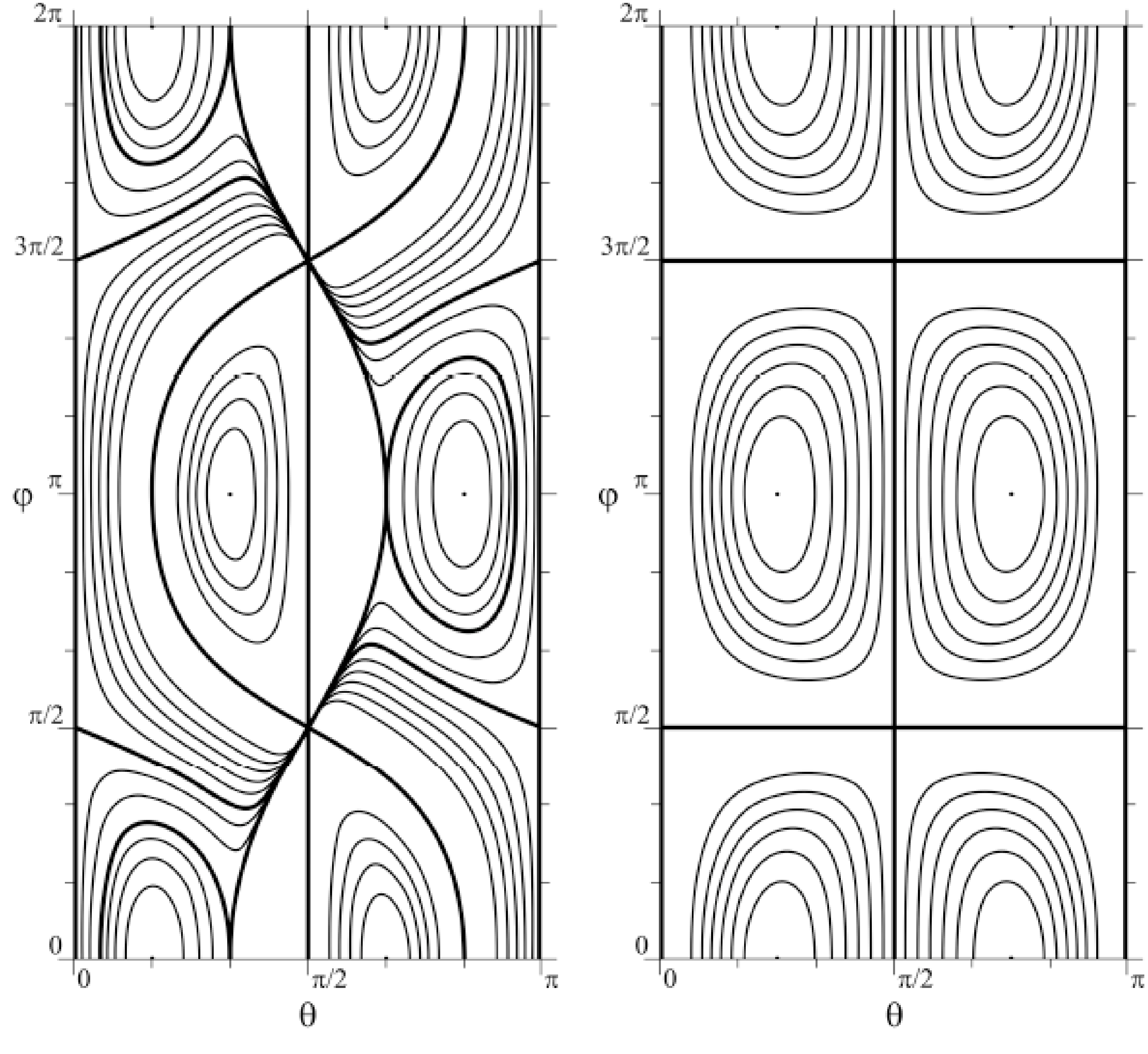
where

$$\xi = kR^2 \frac{m}{r^3} \left[(\cos\theta_m - \cos\theta \cos\theta') \left(\frac{r^2}{R^2} - 1 \right) + \frac{2}{3} \cos\theta_m \right].$$

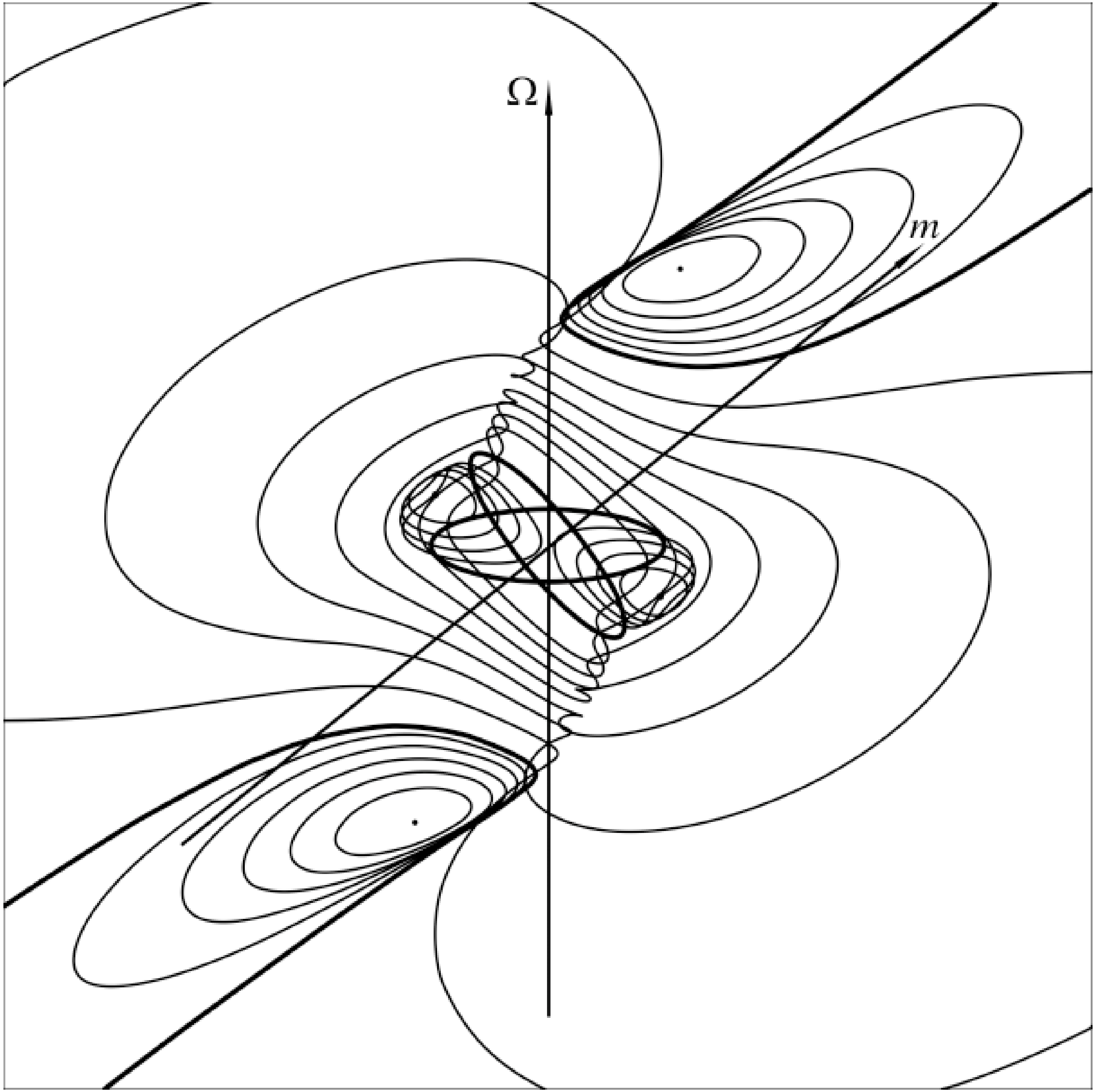
This potential is the integral of motion

$$\xi = \mathfrak{C}_0$$

The trajectory of a particle is the intersection of the FFS and the equipotential surface



2π
3π/2
π
π/2
0
φ
0
π/2
π
θ



Ω
m

Plasma accumulation

Proper electric field of the layer

$$E_l = 4\pi\alpha \int_0^z \rho_e(z')\, dz'$$

$$\cos\psi = \frac{\mathbf{B}\cdot\nabla(\mathbf{E}\cdot\mathbf{B})}{B\,|\nabla(\mathbf{E}\cdot\mathbf{B})|}$$

For arbitrary l we have

$$4\pi\alpha\cos\psi \int_0^{l\cos\psi} \rho_e(z')\, dz' = \omega^2 l.$$

Therefore, the proper electric field is constant (does not depend on l):

$$\rho_e = \frac{\omega^2}{4\pi\alpha\cos^2\psi}.$$

Plasma accumulation

A plasma is completely charge-separated:

$$n_e = |\rho_e|.$$

The equation for the thickness of charged layer:

$$\frac{\partial h}{\partial t} = \frac{j}{n_e}.$$

Time of filling the magnetosphere with plasma

$$\tau_f = \frac{R n_{GJ}}{j}.$$

The diffuse background of galactic photons with the energy larger than 1 MeV

$$10^{-3}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$$

$$j_{ph}/(c n_{GJ}) \simeq 10^{-25}$$

But the characteristic photon mean free path in a polar magnetosphere ~100 m.

$$\exp 100 \simeq 10^{43}.$$

Conclusion

Far from the FFS the particles move virtually along the magnetic field lines. Their energy is determined from the balance between the power of accelerated electric field forces and the curvature radiation intensity.

When intersecting the FFS, the particles oscillate ultrarelativistically, their energy decays first linearly, then in a power-mode. When the curvature radiation intensity becomes less than the bremsstrahlung intensity, the decay is exponential.

Simultaneously with oscillation the particles move along closed FFS trajectories. After capture on the FFS the particles cannot escape.

Capturing particles leads to the monotonous increase in the thickness of plasma charge-separated layer formed in the vicinity of the FFS

Smallness of the cosmic gamma-ray flux can be compensated by the efficient pair creation and not prevent from rapid filling the magnetosphere with plasma.

Thank you

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Astro Conference Hall
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