VIABLE f(R) MODELS OF INFLATION AND DARK ENERGY

$$S = \frac{1}{16\pi G} \int f(R) \sqrt{-g}\, d^4x + S_m$$

$$f(R) = R + F(R) \qquad R \equiv R_\mu^\mu$$

$$R_\mu^\nu - \frac{1}{2}\delta_\mu^\nu R = -8\pi G \left(T_{\mu(m)}^\nu + T_{\mu(DE)}^\nu \right)$$

$$8\pi G\, T_{\mu(DE)}^\nu \equiv F'(R) R_\mu^\nu - \frac{1}{2} F(R) \delta_\mu^\nu$$

$$+ \left(\nabla_\mu \nabla^\nu - \delta_\mu^\nu \nabla_\rho \nabla^\rho \right) F'(R)$$

Particle content: graviton + massive scalar particle ($M^2 = \frac{1}{3f''(R)}$)

(dubbed "scalaron" in A. S., 1980)

Stability conditions:

1. $f' > 0$ graviton is not a ghost
2. $f'' > 0$ scalaron is not a tachyon

imposed for $R \geq R_{now}$ at least

(i.e. during the whole evolution of the Universe)

POSSIBLE MICROSCOPIC ORIGIN OF $f(R)$ GRAVITY

1. Vacuum polarization in curved space-time

2. Reduction to 4-D from curved $(4+n)$-D space-time

3. Limiting case of scalar-tensor gravity

Example: $\mathcal{L} = \xi R \varphi^2 + \frac{1}{2} \varphi_{,\mu} \varphi^{,\mu} - \frac{\lambda \varphi^4}{4}$

reduces to $\mathcal{L} = \frac{M_p^2}{2}\left(R + \frac{R^2}{6M^2}\right)$ in the slow-rolling regime during inflation if $\xi \gg 1$

4. Emergent gravity

F. R. Klinkhamer & G. E. Volovik,
Pisma v ZhETF 88, 339 (2008)
arXiv: 0807.3896

Violation of these conditions is undesirable from the classical point of view, too!

$f'(R_0) = 0$ - instant loss of homogeneity and isotropy

$f''(R_0) = 0$ - weak singularity

$$R(t) = R_0 + O(\sqrt{t})$$
$$a(t) = a_0 + a_1 t + a_2 t^2 + O(t^{5/2})$$

③ Existense of the Newtonian regime

$$(\Delta\varphi = 4\pi G\rho)$$

$$|F| \ll R, \quad |F'(R)| \ll 1, \quad R|F''(R)| \ll 1$$

for $R_{now} \ll R$ (at up to some very large R)

$$\longleftrightarrow$$

De Sitter regime

$$Rf' = 2f$$

Stable if

$$f'(R_1) > R_1 f''(R_1)$$

$$\longleftrightarrow$$

Equivalent to $\omega_{BD} = 0$ scalar-tensor gravity

Use for inflation

$$f(R) = R + \frac{R^2}{6M^2}$$ (+ small non-local terms)

AS, 1980

Internally self-consistent inflationary model with slow-roll decay, a graceful exit to the subsequent RD FRW stage (through an indermediate matter-dominated stage) and sufficiently effective reheating

$\tau \sim M_{Pl}^2 / M^3$ $\quad N \sim 50$

Remains viable

$$M = 3.0 \times 10^{-6} (N/50)^{-1} M_{Pl}$$

$$n_s = 1 - \frac{2}{N} = 0.96$$ for $N = 50$

$$r = \frac{12}{N^2} = 4.8 \times 10^{-3} (N/50)^{-2}$$

Eksp. : $\overline{n_s} = 0.96 \pm 0.014$, $r < 0.20$

OTHER VIABLE $f(R)$ INFLATIONARY MODELS

1. "Chaotic" type: inflation occurs over a large range of R

$$f(R) = R^2 \psi(R)$$

$$\left|\frac{\psi'}{\psi}\right| \ll \frac{1}{R}, \quad \left|\frac{\psi''}{\psi}\right| \ll \frac{1}{R^2}$$

2. "New inflationary" type: inflation occurs around $R = R_0$

$$f'(R_0) \approx 2f(R_0)/R_0$$

$$f''(R_0) \approx 2f(R_0)/R_0^2$$

In both cases: $f(R)$ close to $\frac{R^2}{6M^2}$ over some range of R

Use for DE

$F(R) \propto R^{-n}$ for $R \to 0$

Does not work for many reasons

Viable model – regular at $R=0$

$$f(R) = R + \lambda R_0 \left(\frac{1}{\left(1+\frac{R^2}{R_0^2}\right)^n} - 1 \right)$$

AS, JETP Lett. 86, 157 (2007)
arXiv: 0706.2041 [astro-ph]

or even

$$f(R) = R - \lambda R_0 \tanh^2 \frac{R}{R_0}$$

$f(0) = 0$ – 'disappearing' cosmological constant in flat space-time

Induced Λ at high curvatures:

$$\Lambda_\infty \equiv -\frac{1}{2} F(\infty) = \frac{\lambda R_0}{2}$$

Observational restrictions

1. Cosmology

Anomalous growth of non-relativistic matter perturbations in the regime

$k \gg M(R)\, a$

$G_{eff} = 4G/3f'(R) \simeq \frac{4G}{3}$

$\left(\frac{\delta\rho}{\rho}\right)_m \propto t^{\frac{\sqrt{33}-1}{6}}$ (instead of $\propto t^{2/3}$)

Results in the apparent mismatch

$$\Delta n_s = n_s^{(gal)} - n_s^{(CMB)} = \frac{\sqrt{33}-5}{2(3n+2)}$$

$\Delta n_s < 0.05 \longrightarrow n \geq 2$

2. Laboratory and Solar system tests

$M(R)\, L \gg 1$ with $R = 8\pi G T_m = 8\pi G \rho_m$

otherwise, $\gamma_{PN} = \frac{1}{2}$ and the 'fifth' force appears

$M(R(\rho_m)) \propto \rho_m^{n+1}$ $\quad (R = 8\pi G \rho_m)$

$n \geq 2$ is sufficient for all tests

Structure of corrections at the matter-dominated and earlier stages

$$R = R^{(0)} + \delta R_{ind} + \delta R_{osc}$$

$$R^{(0)} = 8\pi G T_m \propto a^{-3}$$

$$\delta R_{ind} = \left(R F'(R) - 2F(R) - 3 \nabla_\mu \nabla^\mu F'(R) \right)_{R=R^{(0)}}$$

$R \gg R_0$: $\delta R_{ind} \simeq const = -2F(\infty) = 4\Lambda(\infty)$

No Dolgov-Kawasaki instability

$$\delta R_{osc} \propto \begin{cases} t^{-3n-4} \sin(const \cdot t^{-2n-1}) & \text{MD} \\ t^{-\frac{3n}{4}-3} \sin(const \cdot t^{-(3n+1)/2}) & \text{RD} \end{cases}$$

$\frac{\delta a}{a}$ is small but $\frac{\delta R_{osc}}{R^{(0)}}$ diverges for $t \to 0$

δR_{osc} should be very small just from beginning – problem for those $F(R)$ models which do not let R become negative (due to the crossing of the $F''(R) = 0$ point)

"Scalaron overproduction" problem

"Big Boost" SINGULARITY WITH $R \to \infty$ AND ITS ELIMINATION

If $F(R) \to 0$ at $R \to \infty$, then a new generic "Big Boost" singularity can arise:

$F(R) \propto R^{-2n}$; $F''(\infty) = 0$

$a = a_0 + a_1(t - t_0) + a_2|t - t_0|^k$, $1 < k = \frac{2n+1}{n+1} < 2$

$R \propto |t - t_0|^{k-2} > 0$

Elimination:

add $\frac{R^2}{6M^2}$ to $F(R)$ $\qquad M^2(\infty) = M$

Additional advantages:

1. No unlimited growth of $M(R)$
2. A toy "UV-completion" – further radiative corrections are logarithmic only
3. A possibility to unify inflation and present dark energy in one $F(R)$ model if $M = 3 \cdot 10^{-6} M_{Pl}$

$\tau_{dec} \sim \frac{M_{Pl}^2}{M^3} \longrightarrow$ $M > 10^4\,\mathrm{GeV}$ for scalarons to decay before BBN

CONCLUSIONS FOR $f(R)$ MODELS

1. With a regular $f(R)$ satisfying

$f'(R) > 0$, $f''(R) > 0$ for all R

$|f - R| \ll R$, $|f' - 1| \ll 1$, $R|f''| \ll 1$ for $R_0 \ll R \ll M^2$ with $M \gtrsim 10^4$ GeV

$f(R) \propto R^2$ for $R \to \infty$,

it is possible to construct viable models of DE, satisfying all existing cosmological, Solar system and laboratory data, and distinguishable from ΛCDM

2. Further unification of primordial DE (producing inflation) and present DE is possible for the specific choice of M: $M \approx 3 \cdot 10^{-6} M_{Pl}$

3. The most critical test of these DE models: anomalous growth of scalar perturbations at recent time ($z \sim 1-3$ for $L = 8h^{-1}$ Mpc)