Is There 2D Anderson Transition?

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According to one-parameter scaling theory

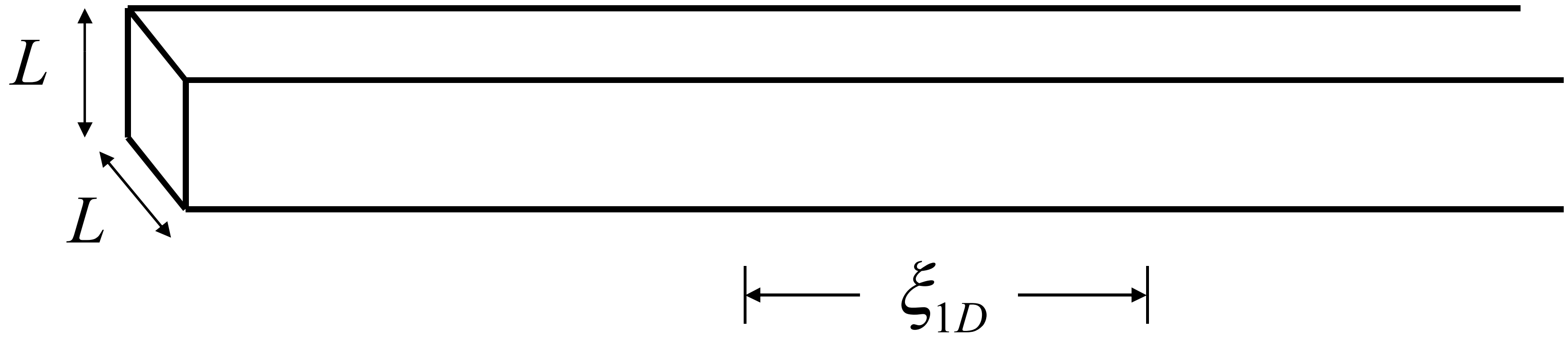
E.Abrahams, P.W.Anderson
D.E.Licciardello, T.V.Ramakrishnan,
Phys.Rev.Lett., 42, 673 (1979)

2D Anderson transition is absent.

Nevertheless, 2D metal-insulator transition is observed experimentally

S.V.Kravchenko, G.V.Kravchenko, J.E.Furneaux,
V.M.Pudalov, M.D.Jorio,
Phys.Rev.B 50, 8039 (1994)

Finite-size scaling

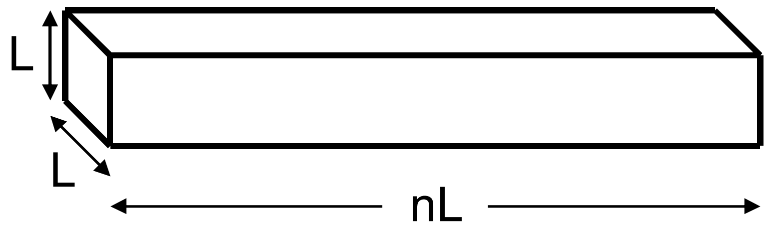


T>T$_C$ (paramagnetic phase):

$$\xi_{1D} \sim \xi \qquad (L \to \infty)$$

T<T$_C$ (ferromagnetic phase):

$$\frac{\xi_{1D}}{L} \to \infty \qquad (L \to \infty)$$

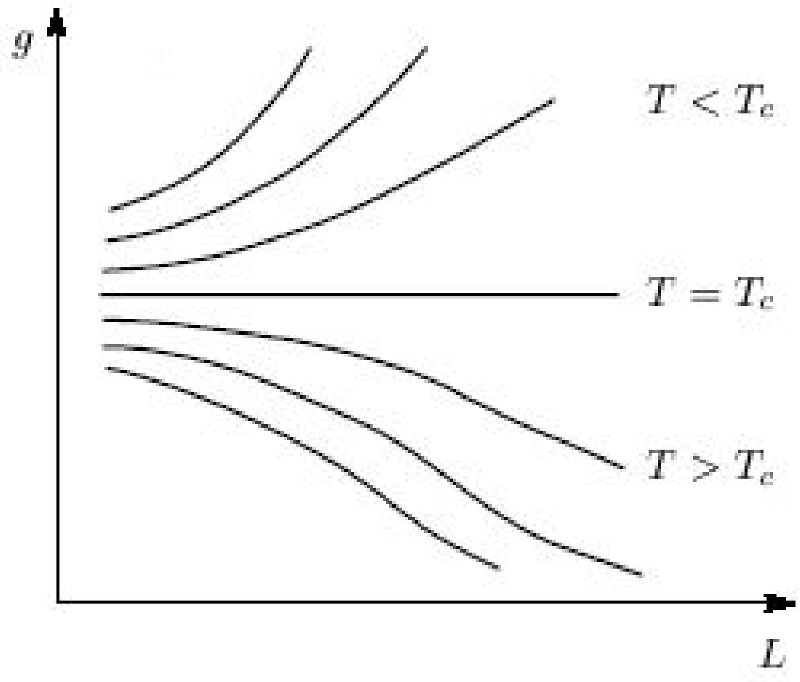


Proof by contradiction:

$$\frac{\xi_{1D}}{L} \leq C \qquad n >> C$$

Scaling parameter

$$g(L) = \frac{\xi_{1D}}{L}$$



One-parameter scaling

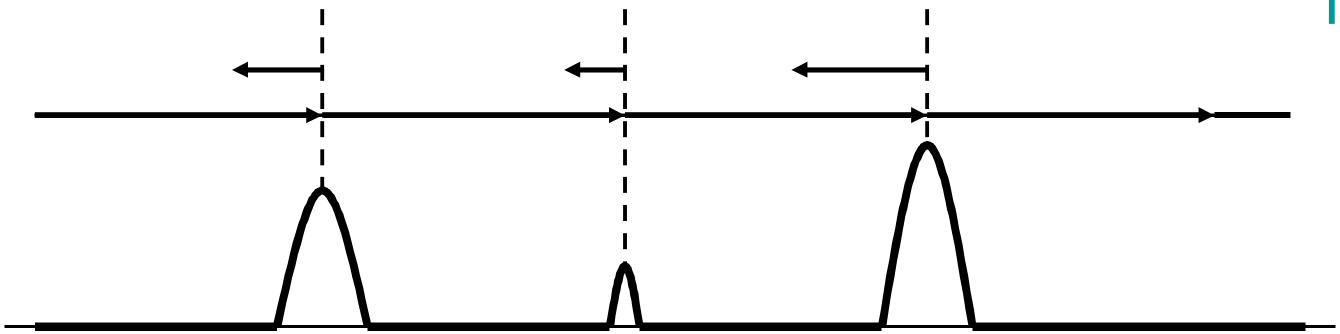
$$g = F\left(\frac{L}{\xi}\right)$$

Estimate through the minimal Lyapunov exponent

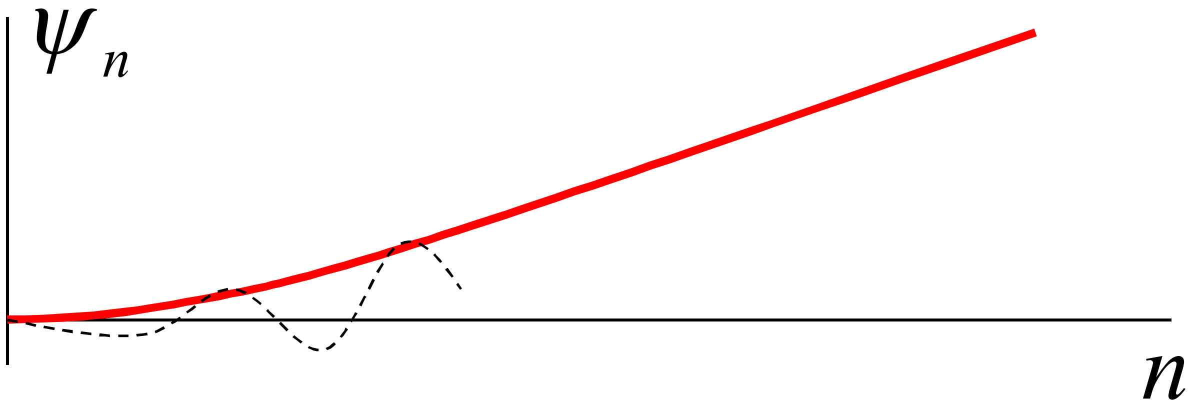
$$\xi_{1D} \sim \frac{1}{\gamma_{\min}}$$

Lyapunov exponents

N.F.Mott, 1961

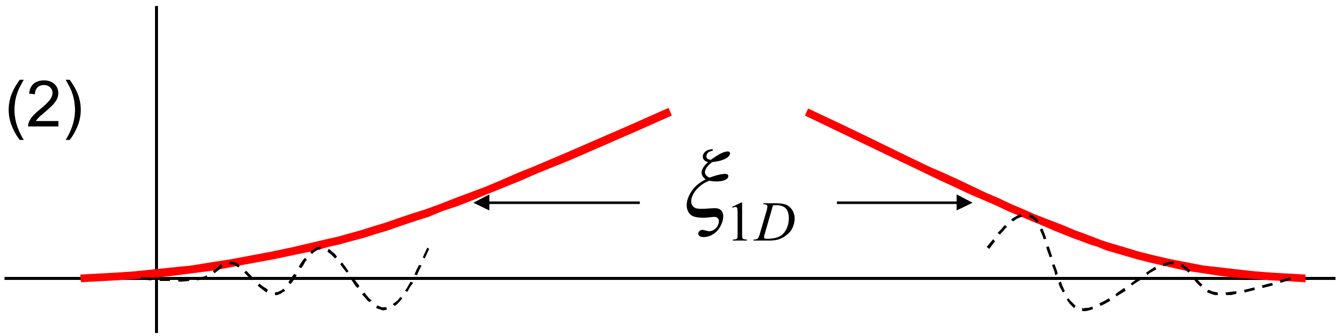


(1)

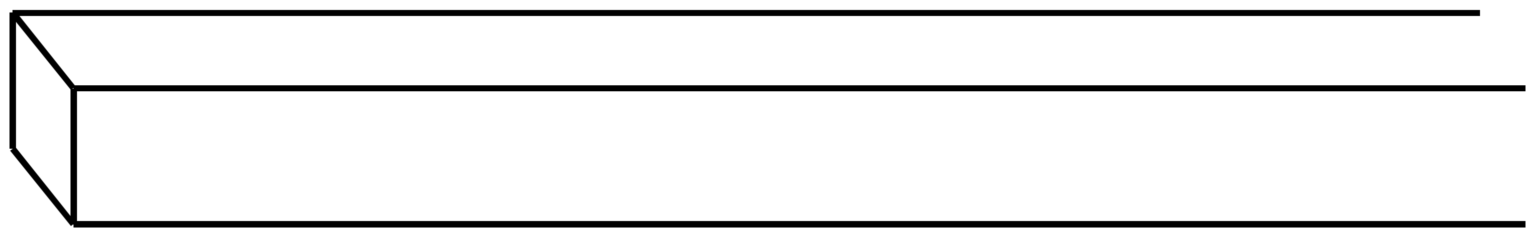


$$\psi_n \sim e^{\gamma n}$$

(2)



$$\xi_{1D} \sim \frac{1}{\gamma}$$



$$\psi_n(r_\perp) = A_1 h_n^{(1)}(r_\perp)\, e^{\gamma_1 n} + A_2 h_n^{(2)}(r_\perp)\, e^{\gamma_2 n} + \ldots + A_m h_n^{(m)}(r_\perp)\, e^{\gamma_m n}.$$

$\gamma_{\min}$

For average quantities

$$\langle \psi_n(r_\perp) \rangle \sim 1$$

$$\langle \psi_n^2(r_\perp) \rangle = B_1(r_\perp)\, e^{\beta_1 n} + B_2(r_\perp)\, e^{\beta_2 n} + \ldots + B_m(r_\perp)\, e^{\beta_m n}$$

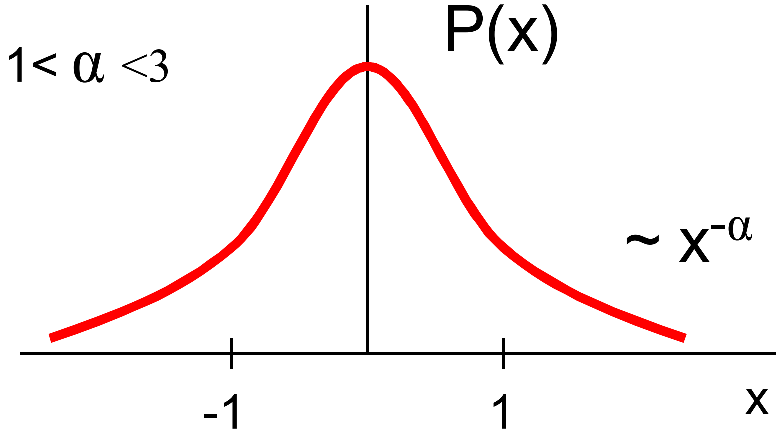
Correspondence of two decompositions

$$B_s(r_\perp)\ e^{\beta_s n} = \left\langle \left(A_s h_n^{(s)}(r_\perp)\ e^{\gamma_s n} \right)^2 \right\rangle \quad \rightarrow \quad \beta_s \geq 2\gamma_s$$

From $\langle x \rangle = 0$, $\langle x^2 \rangle = \sigma^2$ one is tempted to derive $x \sim \sigma$ for the typical value; in fact only $|x| \lesssim \sigma$ is valid

Chebyshev inequality

$$P\{ |x| > x_0 \} < \frac{\sigma^2}{x_0^2}$$



$$x \sim 1, \qquad \langle x^2 \rangle = \sigma^2 = \infty$$

Logarithmically normal distribution

In the 1D case

$$\psi_n \sim e^{\gamma n}, \quad \langle \psi_n \rangle \sim 1, \quad \langle \psi_n^2 \rangle \sim e^{\beta n}.$$

one has distribution

$$P(\tau) \sim \exp\left\{-\frac{(\tau - an)^2}{2bn}\right\}, \qquad \tau = \ln|\psi_n|$$

and

$$\psi_n \sim e^{an}, \quad \langle \psi_n^2 \rangle \sim e^{(2a+2b)n}$$

Here

$$\left.\begin{array}{ll} a = b, & W \to 0 \\ a >> b, & W \to \infty \end{array}\right\} \quad \to \quad \gamma \sim \beta$$

In the quasi-1D case:

Weak disorder: A. M. S. Macedo and J. T. Chalker, Phys. Rev. B **46**, 14985 (1992); M. Caselle, Phys. Rev. Lett. **74**, 2776 (1995); C. W. J. Beenakker and B. Rejaei, Phys. Rev. Lett. **71**, 36891 (1993); Phys. Rev. B **49**, 7499 (1994).

Strong disorder: E. Abrahams and M. S. Stephen, J. Phys. C **13**, L377 (1980).

Numerical research: P. Markos and B. Kramer, Philos. Mag. **68**, 357 (1993); P. Markos, J. Phys.: Condens. Matter **7**, 8361 (1995); K. Slevin, Y. Asada, and L. I. Deych, cond-mat/0404530.

Relation between $\gamma_{\min}$ and $\beta_{\min}$

1. Rigorous inequality $\beta_{\min} \geq 2\gamma_{\min}$

2. The order of magnitude relation $\beta_{\min} \sim \gamma_{\min}$ for a typical physical situation

3. Equivalence from viewpoint of one-parameter scaling

$$\frac{1}{\gamma_{\min} L} = F\left(\frac{L}{\xi}\right) \qquad \frac{1}{\beta_{\min} L} = F\left(\frac{L}{\xi}\right)$$

Growth of the second moments

D.J.Thouless, 1974

Cauchy problem for the 1D Anderson model

$$\psi_{n+1} + \psi_{n-1} + V_n \psi_n = E\psi_n$$

where ψ_0 and ψ_1 are fixed. Here V_n are independent quantities, and

$$\langle V_n \rangle = 0, \quad \langle V_n V_{n'} \rangle = W^2 \delta_{nn'}.$$

By direct iterations we have $\psi_2=f(V_1)$, $\psi_3=f(V_1, V_2)$ etc.

$$\langle \psi_{n+1} \rangle = E\langle \psi_n \rangle - \langle \psi_{n-1} \rangle,$$

$$\langle \psi_{n+1}^2 \rangle = (W^2 + E^2)\langle \psi_n^2 \rangle - 2E\langle \psi_n \psi_{n-1} \rangle + \langle \psi_{n-1}^2 \rangle$$

Setting $x_n = \langle \psi_n^2 \rangle$, we have for E=0

$$x_{n+1} = W^2 x_n + x_{n-1} \qquad x_n = \langle \psi_n^2 \rangle \sim e^{\beta n}, \quad 2\sinh\beta = W^2.$$

For $E\neq0$:

$$x_{n+1} = (W^2 + E^2)x_n + x_{n-1} - 2Ey_n, \qquad x_n = \langle \psi_n^2 \rangle,$$

$$y_{n+1} = Ex_n - y_n, \qquad y_n = \langle \psi_n \psi_{n-1} \rangle$$

2D Anderson model

$$\psi_{n+1,m} + \psi_{n-1,m} + \psi_{n,m+1} + \psi_{n,m-1} + V_{n,m}\psi_{n,m} = E\psi_{n,m}$$

Introducing quantities

$$x_{m,m'}(n) \equiv \langle \psi_{n,m}\psi_{n,m'} \rangle,$$
$$y_{m,m'}(n) \equiv \langle \psi_{n,m}\psi_{n-1,m'} \rangle,$$
$$z_{m,m'}(n) \equiv \langle \psi_{n-1,m}\psi_{n,m'} \rangle,$$

we have a set of difference equations (E=0):

$$x_{m,m'}(n+1) = W^2\delta_{m,m'}x_{m,m'}(n) + x_{m+1,m'+1}(n) + x_{m-1,m'+1}(n) + x_{m+1,m'-1}(n) + x_{m-1,m'-1}(n) + x_{m,m'}(n-1) + y_{m+1,m'}(n) + y_{m-1,m'}(n) + z_{m,m'+1}(n) + z_{m,m'-1}(n),$$

$$y_{m,m'}(n+1) = -x_{m+1,m'}(n) - x_{m-1,m'}(n) - z_{m,m'}(n),$$

$$z_{m,m'}(n+1) = -x_{m,m'+1}(n) - x_{m,m'-1}(n) - y_{m,m'}(n).$$

Solution is exponential in n

$$x_{m,m'}(n) = x_{m,m'}e^{\beta n} \quad \text{etc.}$$

Formal substitution

$$x_{m,m'} \equiv \tilde{x}_{m,m'-m} \equiv \tilde{x}_{m,l} \quad \text{etc.,} \qquad l = m' - m$$

gives

$$(e^{\beta} - e^{-\beta})x_{m,l} = W^2\delta_{l,0}x_{m,l} + x_{m+1,l} + x_{m-1,l} + x_{m+1,l-2} + x_{m-1,l+2}$$
$$+ y_{m+1,l-1} + y_{m-1,l+1} + z_{m,l+1} + z_{m,l-1},$$

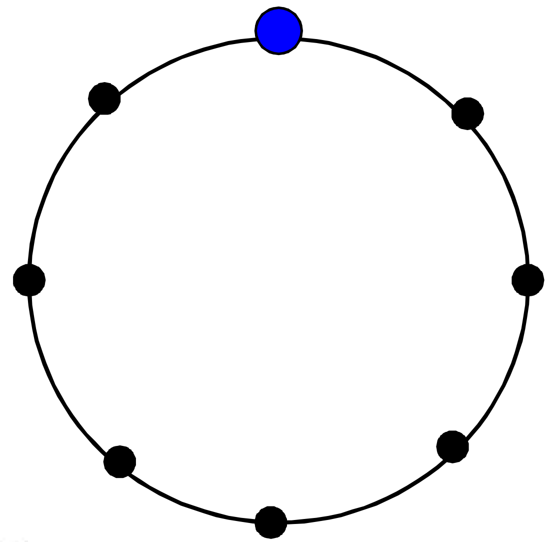
$$e^{\beta}y_{m,l} = -x_{m+1,l-1} - x_{m-1,l+1} - z_{m,l},$$

$$e^{\beta}z_{m,l} = -x_{m,l+1} - x_{m,l-1} - y_{m,l}.$$

Dependence on m is exponential

$$x_{m,l} = x_l e^{ipm} \quad \text{etc.,} \qquad p_s = 2\pi s/L,\ s = 0,\ 1,\ \ldots,\ L-1$$

The problem of one impurity atom in the finite chain



$$x_{l+2}e^{-ip} + x_{l-2}e^{ip} + V\delta_{l,0}x_l = \epsilon x_l, \quad x_{l+L} = x_l,$$

$$\epsilon = 2\cosh\beta, \quad V = \frac{W^2 \sinh\beta}{\cosh\beta - \cos p},$$

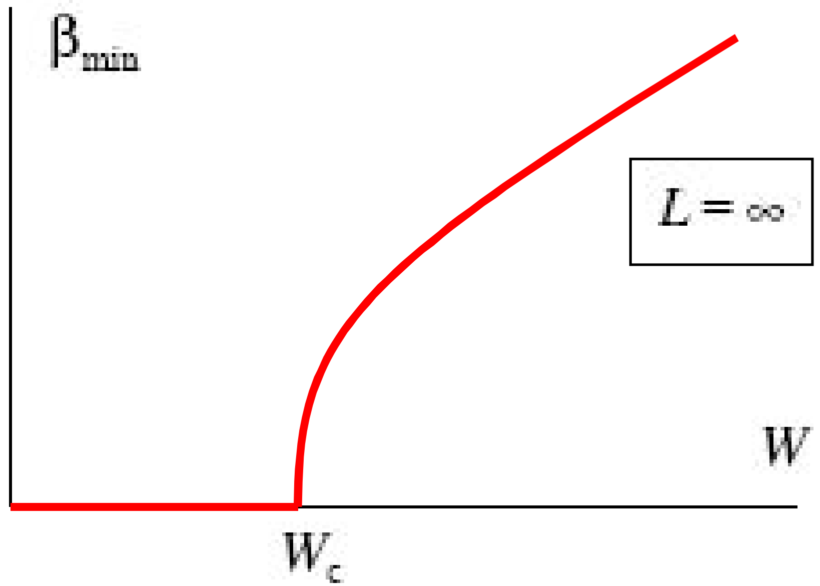
Its spectrum

$$2(\cosh\beta_s - \cos p_s) = W^2 \coth(\beta_s L/2), \qquad p_s = 2\pi s/L$$

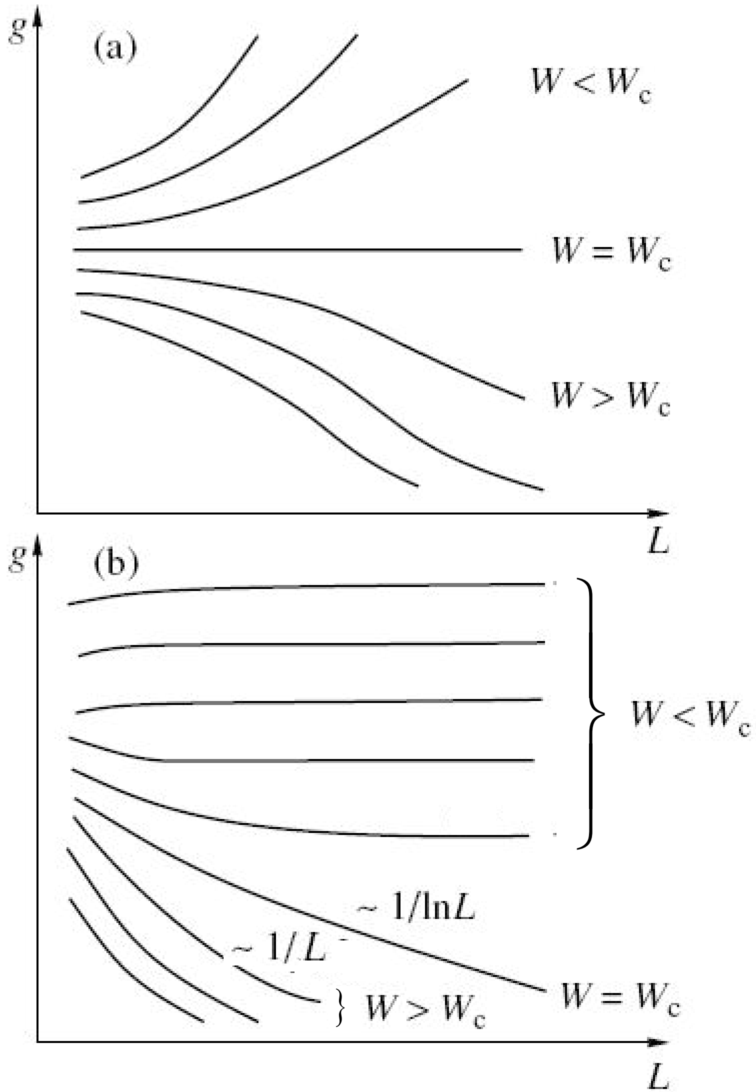
Minimal exponent corresponds to p=π :

$$\beta_{\min} = \begin{cases} \operatorname{arccosh}(W^2/2 - 1), & W^2 > 4 \\ \dfrac{2}{L}\operatorname{arctanh}(W^2/4), & W^2 < 4 \\ \dfrac{2\ln L - 2\ln\ln L + \dots}{L}, & W^2 = 4. \end{cases}$$

Solution is changed qualitatively at $W_c = 2.$



One-parameter scaling picture



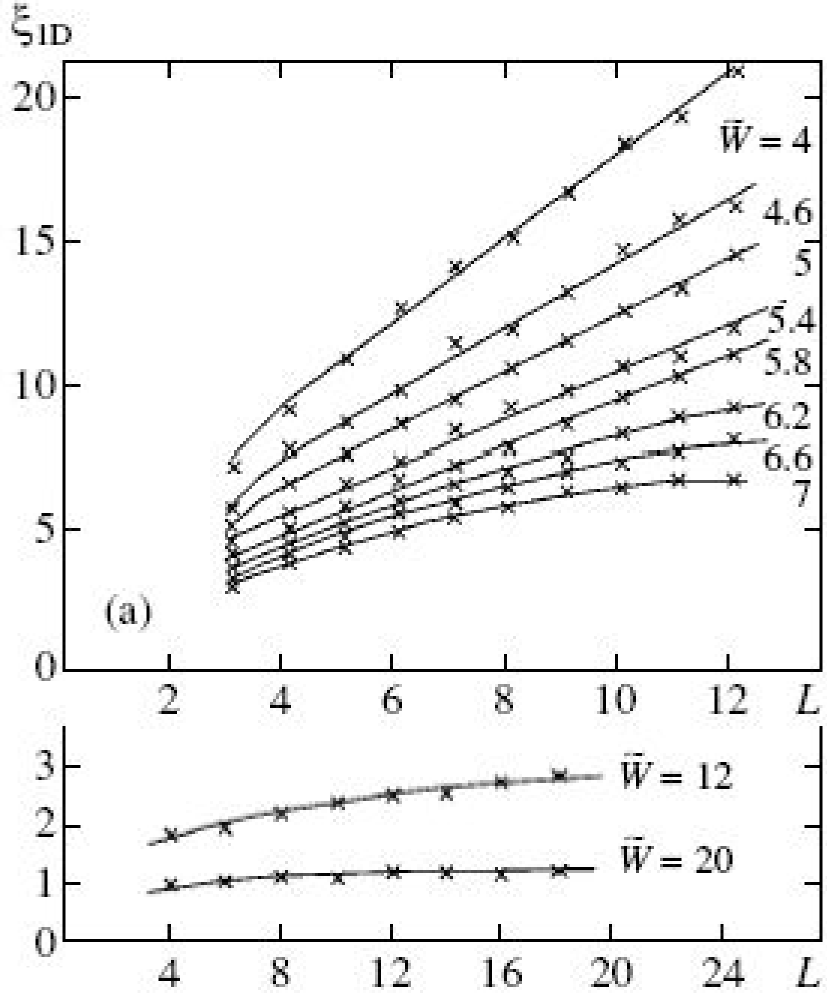
Present results

$$\xi_{1D} \sim 1/\beta_{min}$$

$$g(L) = \frac{\xi_{1D}}{L}$$

(1) Absence of long-range order
(2) Rough violation of scaling

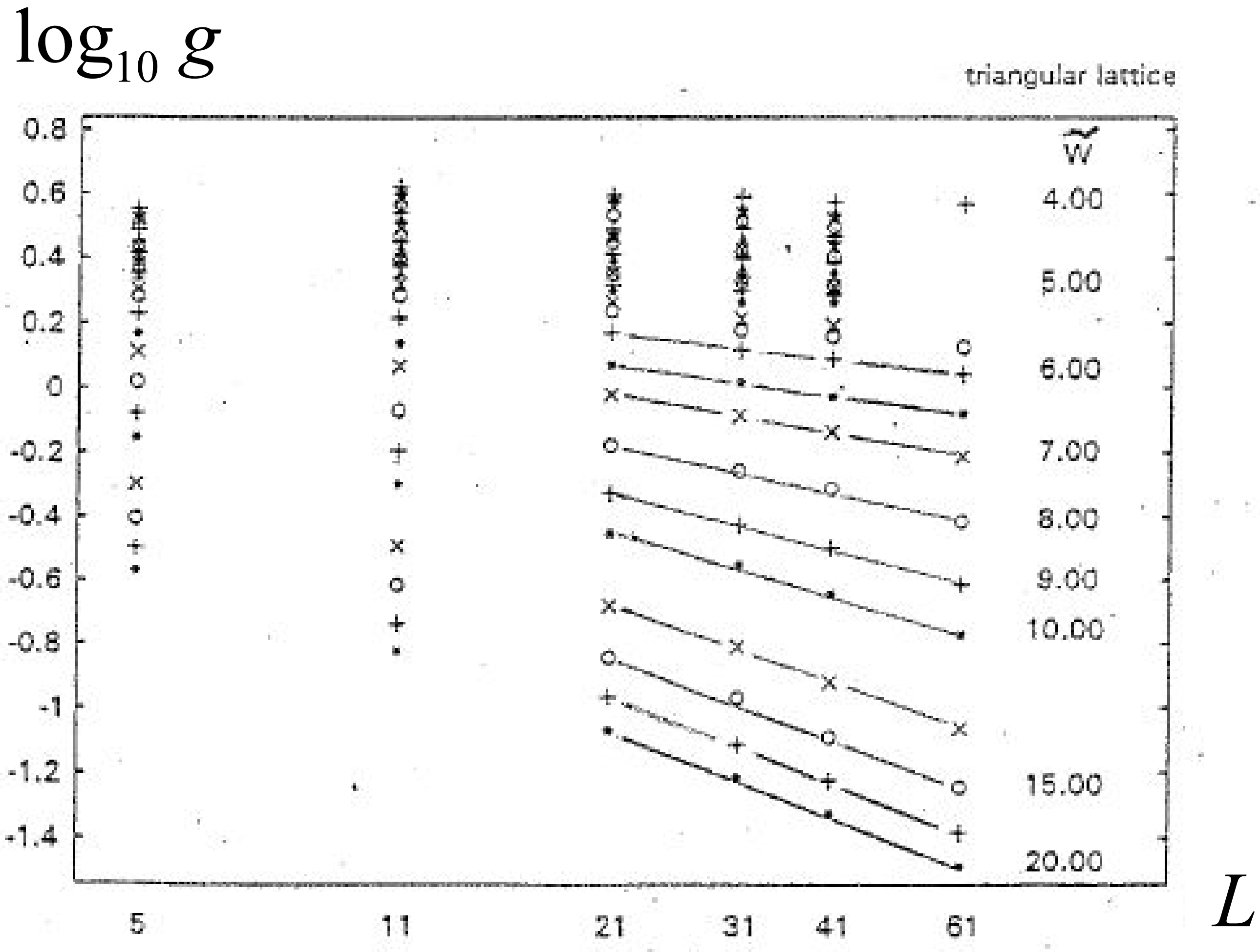
Comparison with numerical research



J.L.Pichard, G.Sarma,
J.Phys. C 14, L617 (1981)

Square distribution
of width $\tilde{W}$

$$\tilde{W} = W\sqrt{12} \qquad \tilde{W}_c = \sqrt{48} = 6.93$$



M.Schreiber, M.Ottomeier, J.Phys.: Cond.Matt. 4, 1959 (1992)

Is there possibility of one-parameter scaling ?

Two interpretations are possible:

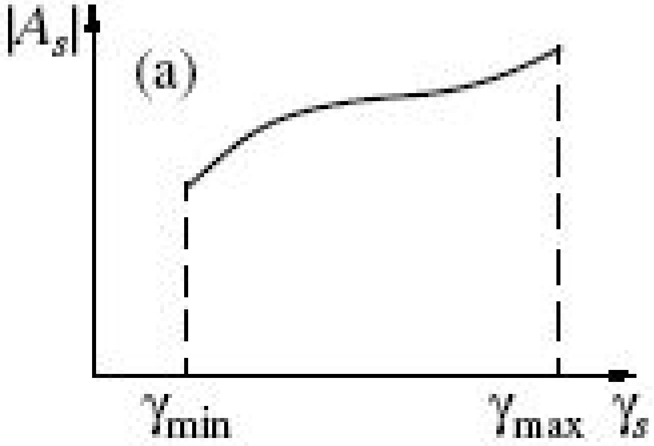
(1) One-parameter scaling hypothesis is fundamentally wrong

(2) Minimal Lyapunov exponent is a bad scaling variable

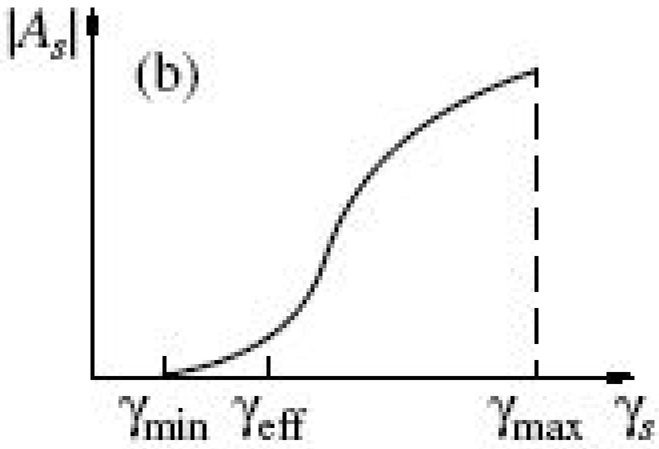
$$\frac{\xi_{1D}}{L} = F\left(\frac{L}{\xi}\right)$$

$$\xi_{1D} \sim \frac{1}{\gamma_{\min}}$$

$$\psi_n(r_\perp) = A_1 h_n^{(1)}(r_\perp) e^{\gamma_1 n} + A_2 h_n^{(2)}(r_\perp) e^{\gamma_2 n} + \ldots + A_m h_n^{(m)}(r_\perp) e^{\gamma_m n}.$$

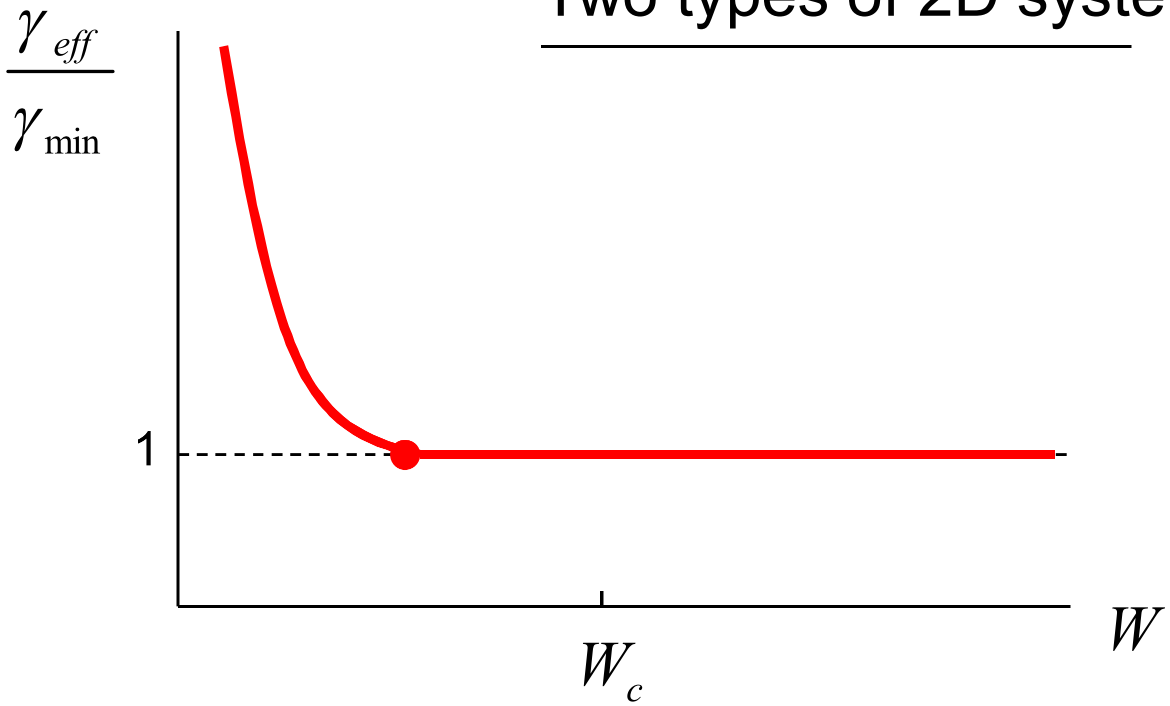


$$\xi_{1D} \sim \frac{1}{\gamma_{eff}}$$

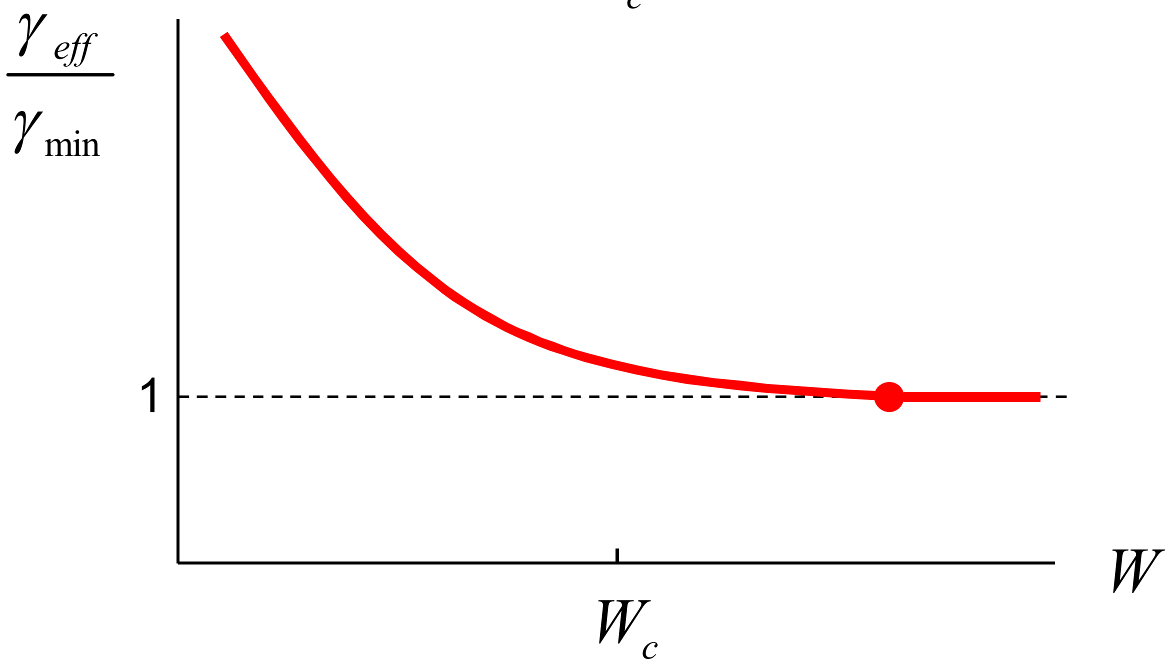


$$\frac{1}{\gamma_{eff} L} = F\left(\frac{L}{\xi}\right)$$

Two types of 2D systems



2D transition survives, but behavior of correllation length is changed



2D transition is absent

E.I.Zavaritskaya, 1980-1990

Conductivity

1. Conductance of the quasi -1D system of length l

$$G(l) \sim \exp\{-2\gamma_{\min} l\}, \qquad l \to \infty$$

Extrapolation to $l \sim L$

$$G(L) \sim \exp\{-2\gamma_{\min} L\} = \begin{cases} \exp\{-const\ L\}, & W > W_c \\ const, & W < W_c \end{cases}$$

and dependence G(L) is determined by the pre-exponential factor.

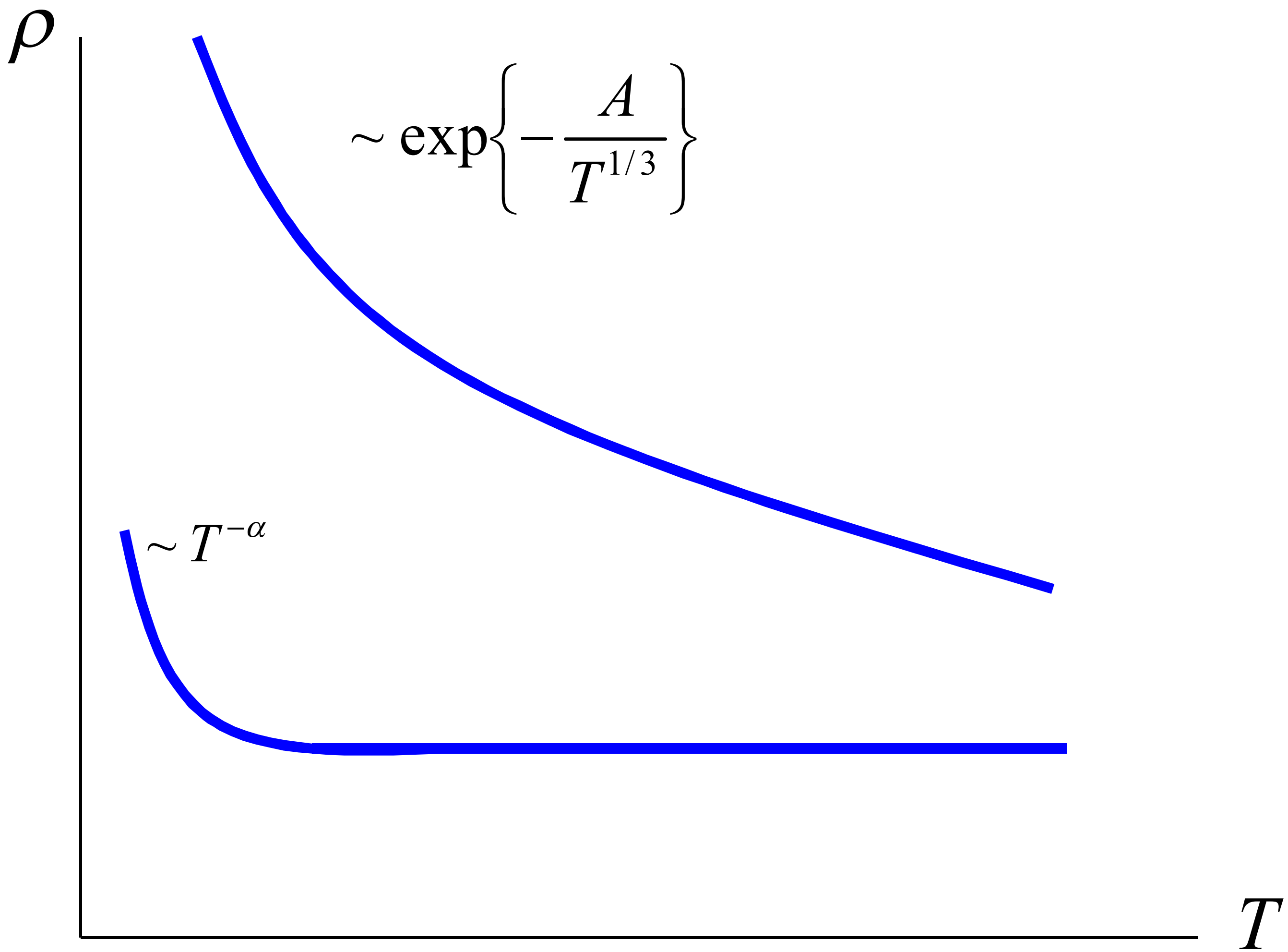
2. Hopping conductivity over power-localized states

$$\sigma(T) \sim T^{4+5\delta}$$

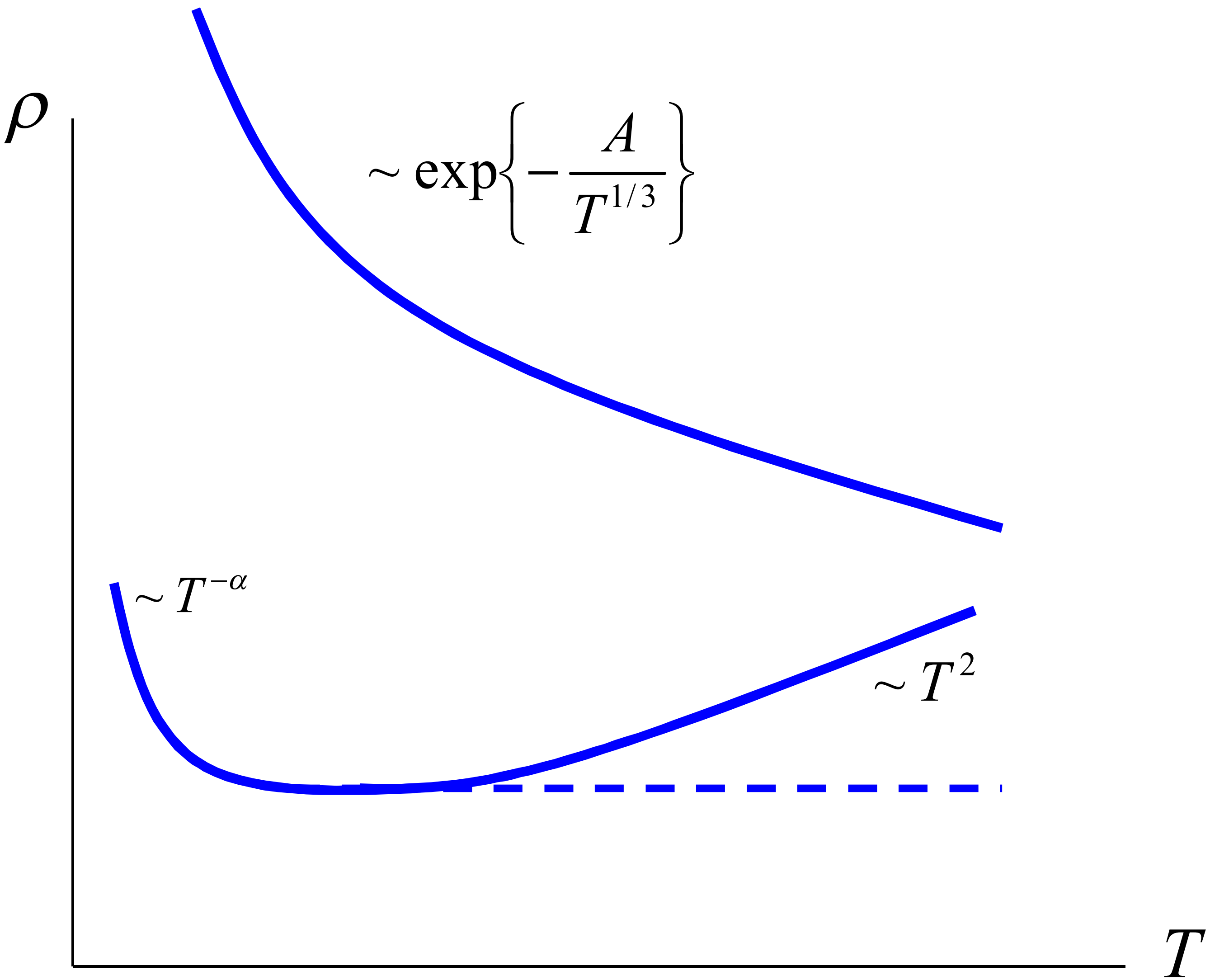
if $\psi(r) \sim r^{\beta}, \qquad \delta = (1+2\beta)^{-1}$

B.J.Last, D.J.Thouless, J.Phys.C 7, 699 (1974)

Experimental consequences

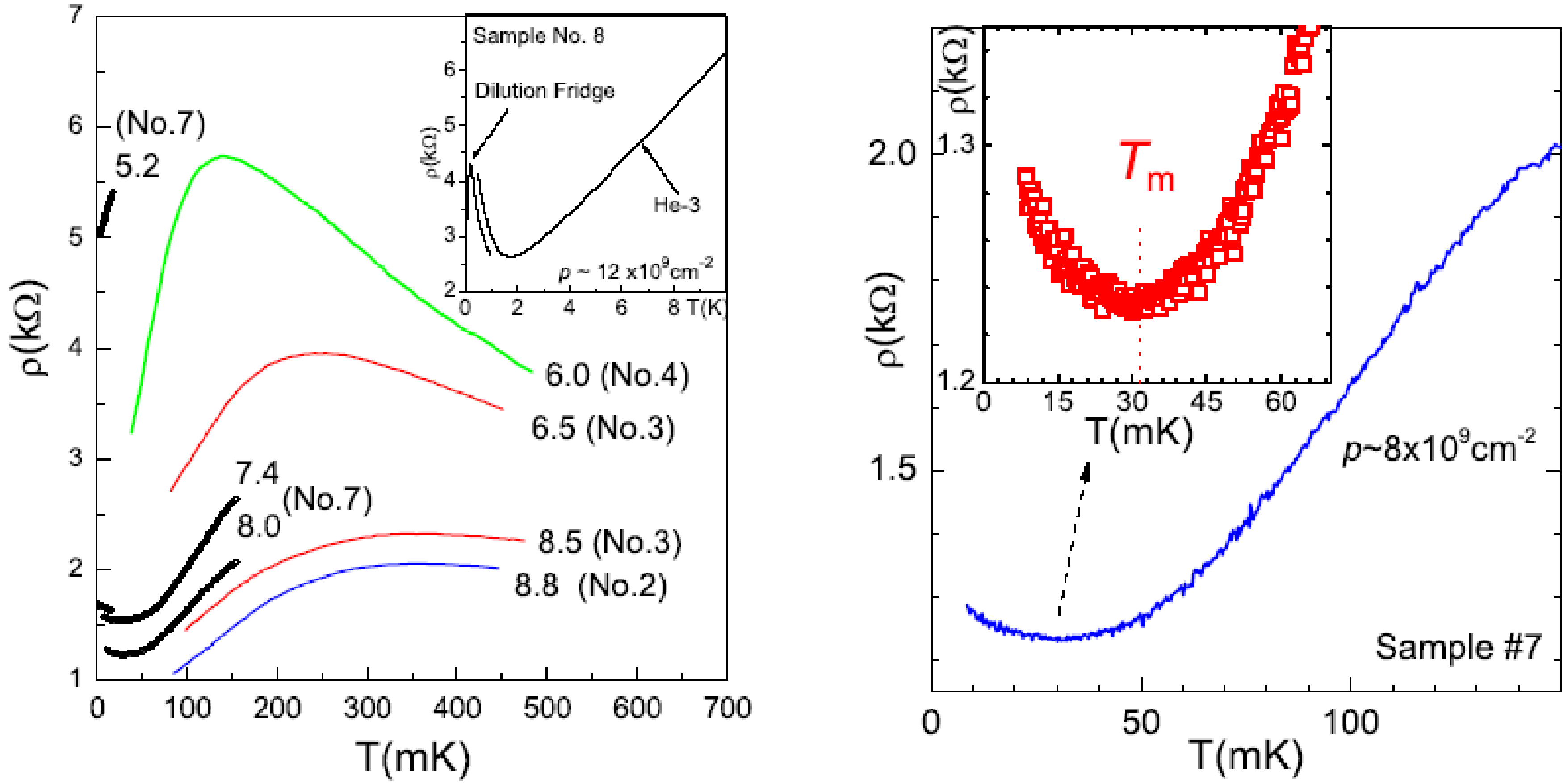


Experimental consequences

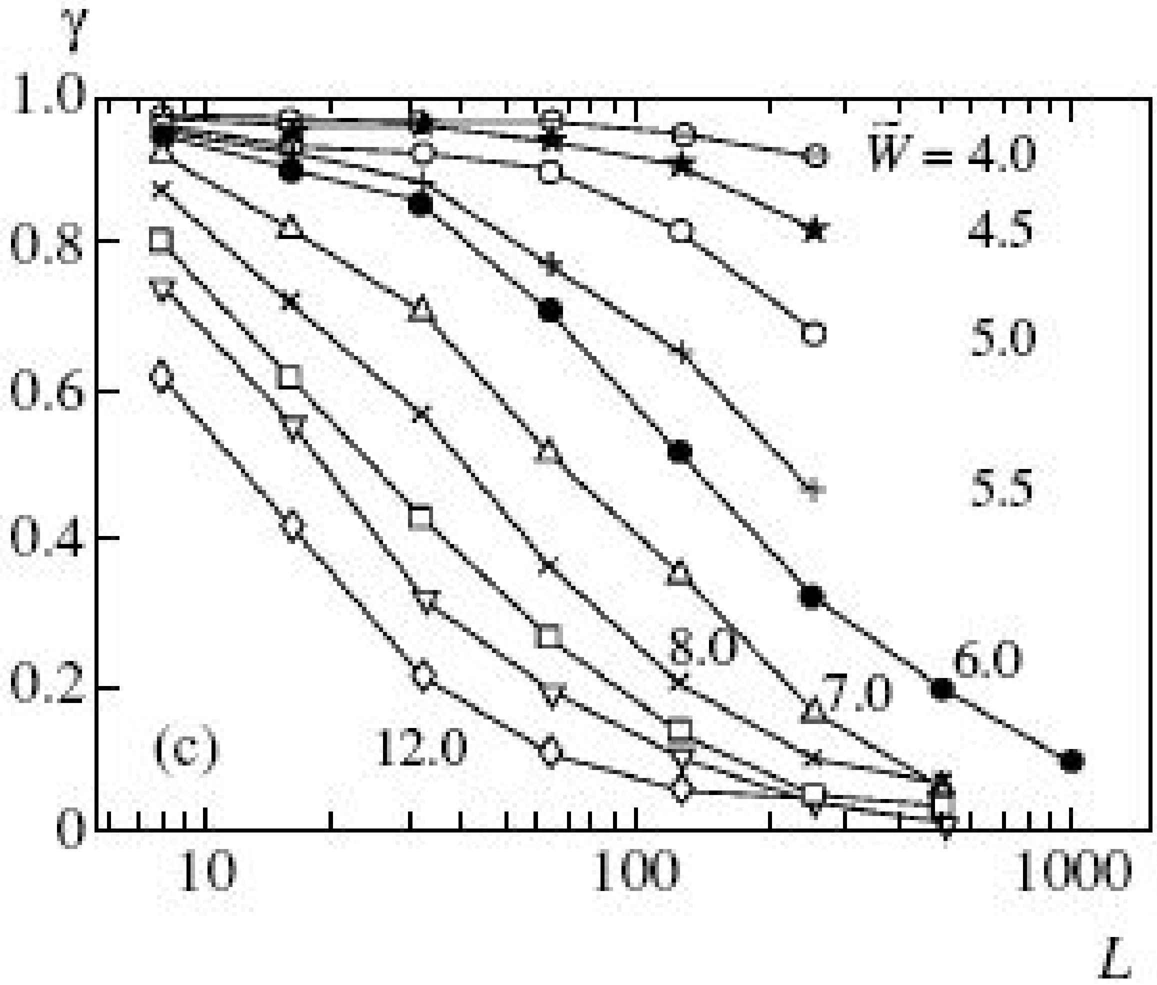


Disappearance of the metal-like behavior in GaAs two-dimensional holes below 30 mK

Jian Huang,[1] J. S. Xia,[2] D. C. Tsui,[3] L. N. Pfeiffer,[4] and K. W. West[4]
[1] PRISM, Princeton University, Princeton, NJ 08544, USA
[2] Department of Physics, University of Florida, Gainesville, FL USA
[3] Department of Electrical Engineering, Princeton University, Princeton, NJ 08544, USA
[4] Bell Labs, Lucent Technologies, Murray Hill, NJ 07974, USA
(Dated: December 20, 2007)

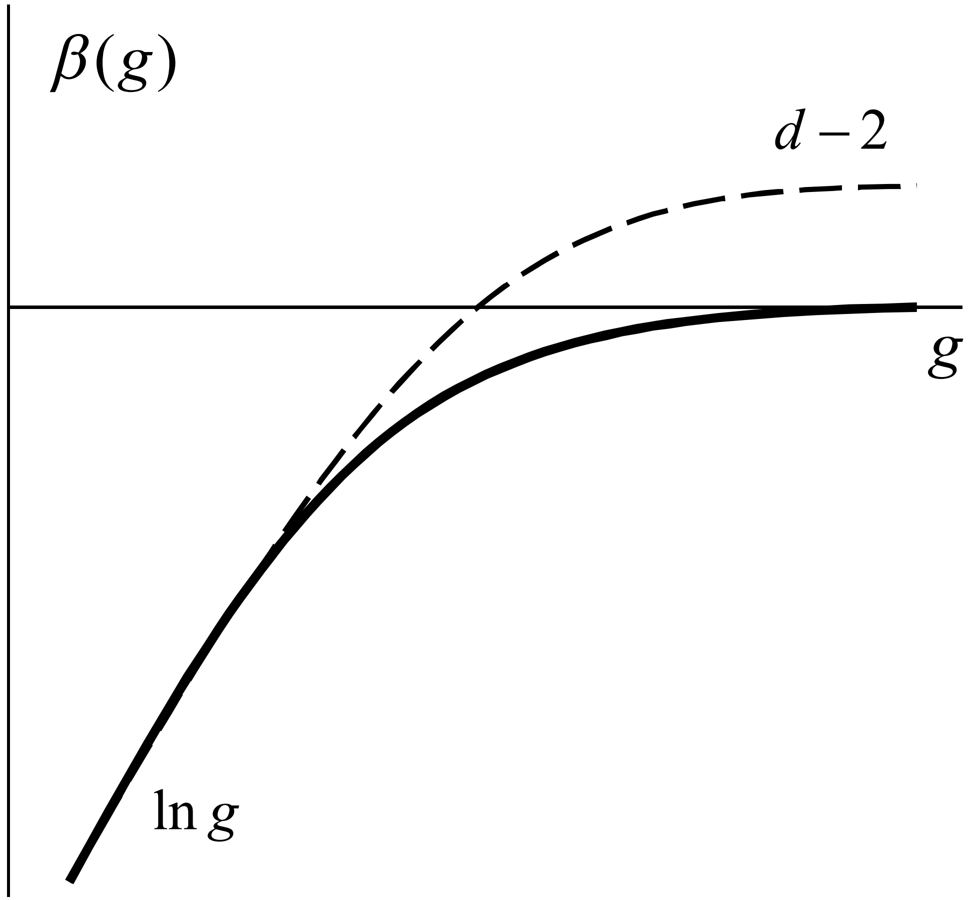


Statistics of levels



I. Kh. Zharekeshev and B. Kramer, Phys. Rev. B **51**, 17 239 (1995).

Correspondence with Abrahams et al



$$g(L) = \frac{G_L}{(e^2/h)}$$

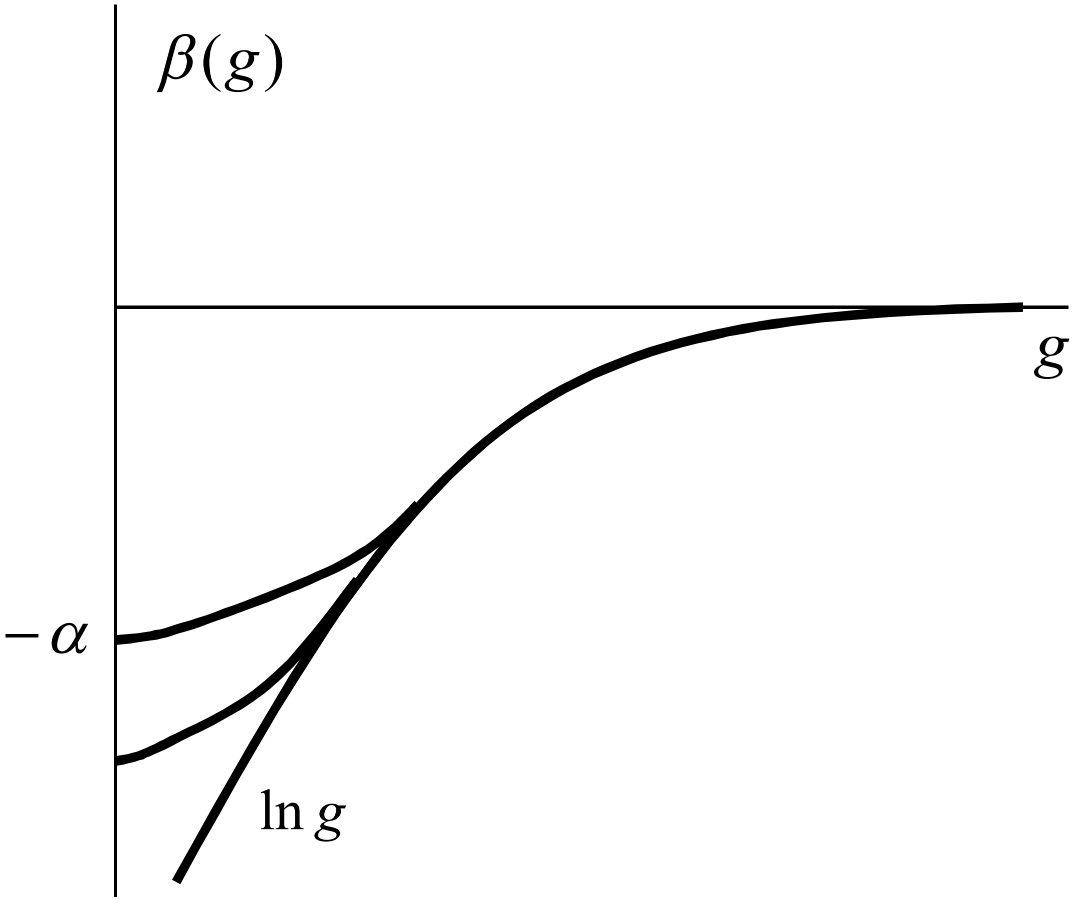
$$g(bL) = F\big(g(L),\ b\big)$$

$$\frac{d\ln g}{d\ln L} = \beta(g)$$

$g >> 1$ $\qquad g = \sigma\ L^{d-2} \quad \rightarrow \quad \beta(g) = (d-2) + \frac{A}{g} \qquad (A<0)$

$g << 1$ $\qquad g \sim \exp(-cL) \quad \rightarrow \quad \beta(g) = \ln g$

Correspondence with Abrahams et al



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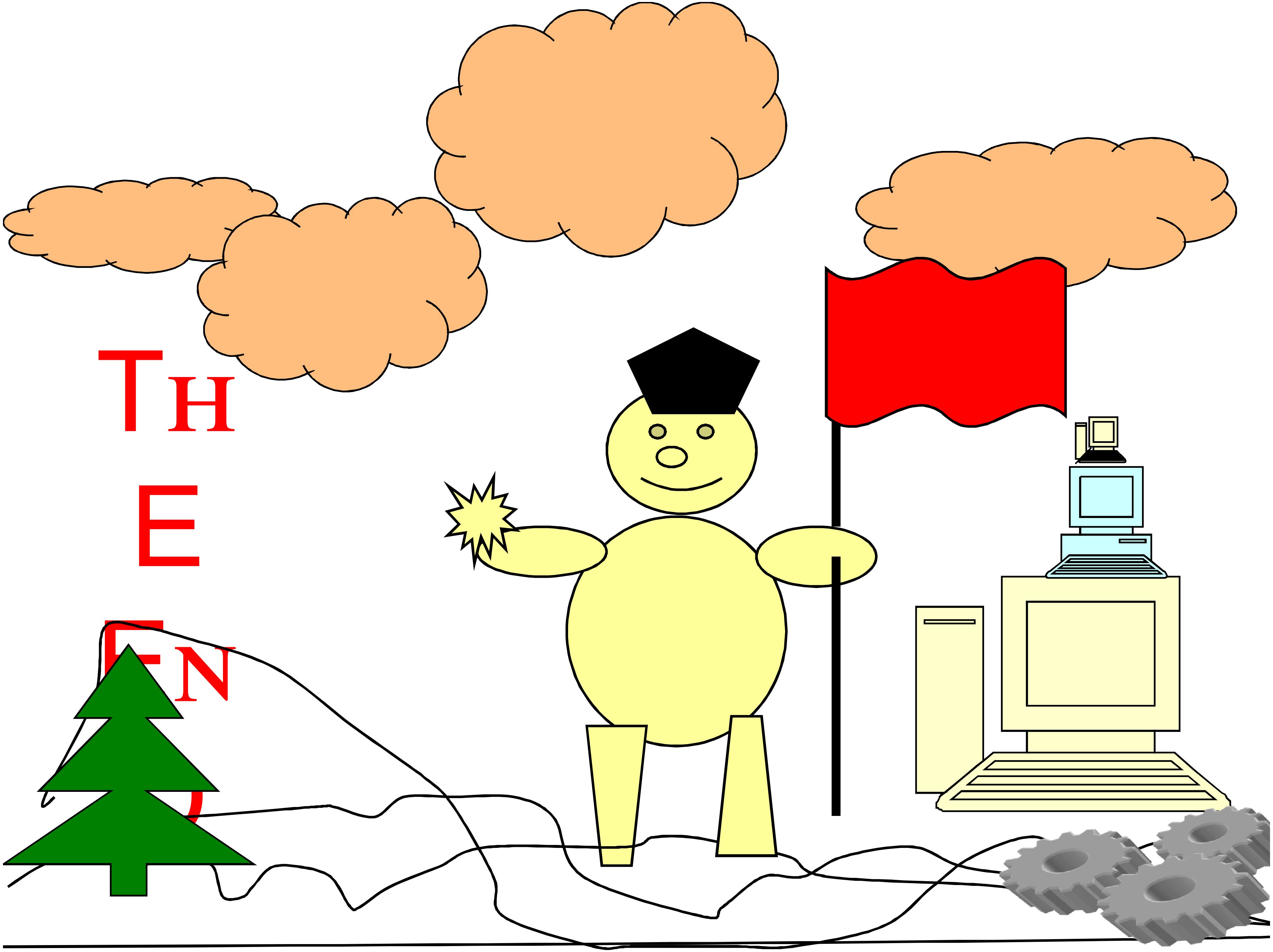
$$\frac{d \ln g}{d \ln L} = \beta(g)$$

$\underline{g >> 1}$ $\quad g = \sigma\ L^{d-2} \quad \rightarrow \quad \beta(g) = (d-2) + \frac{A}{g} \quad (A < 0)$

$\underline{g << 1}$ $\quad g \sim L^{-\alpha} \quad \rightarrow \quad \beta(g) = -\alpha$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