Quasi black holes: Definitions and general properties

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4 papers in PRD
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Usual situation: size approaches gravitational radius, system collapses

Special cases when gravitational radius is approached by sequence of static configurations

Majumdar –Papapetrou systems $p = 0,$

Compact objects: Bonnor stars $\rho = \rho_e$

Sphere of neutral hydrogen lost 10^{-18} of its electrons

Self-gravitating magnetic monopole

Threshold of formation of event horizon. Quasihorizon

Massive charged extremal shells

Different physical systems share common features: geometry of spacetime

behavior of tidal forces

Contents

- Space-time properties: limiting transition. Inside and outside.
- Mass formula: BH and QBH
- Entropy of QBH
- QBHs as mimickers of BHs

Lemos and E. Weinberg 2004

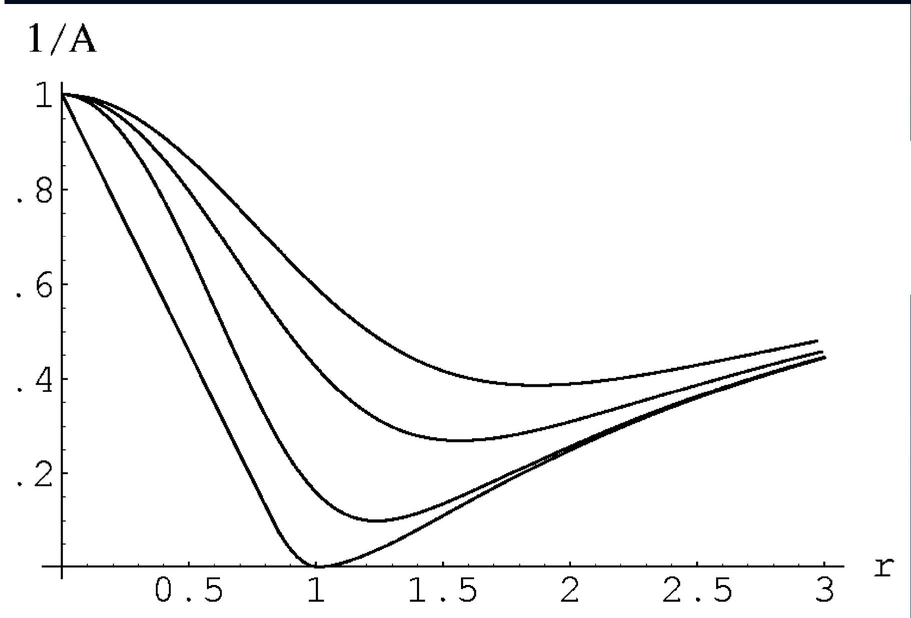


FIG. 1. A plot of $1/A$ as a function of r for $q=1$ and, reading from the top down, $c=0.5$, 0.3, 0.1, 0.001. The emergence of the quasihorizon is quite evident in the $c=0.001$ curve.

$$ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Lue and E. Weinberg 2000

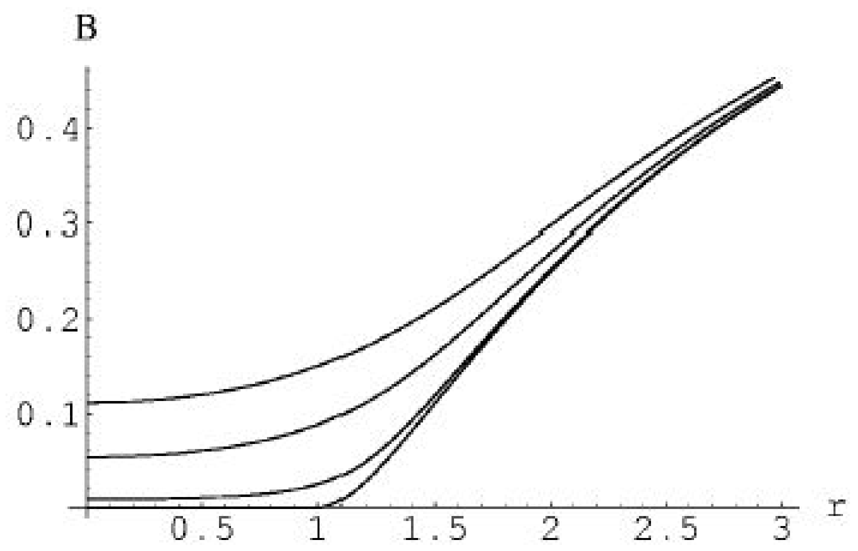
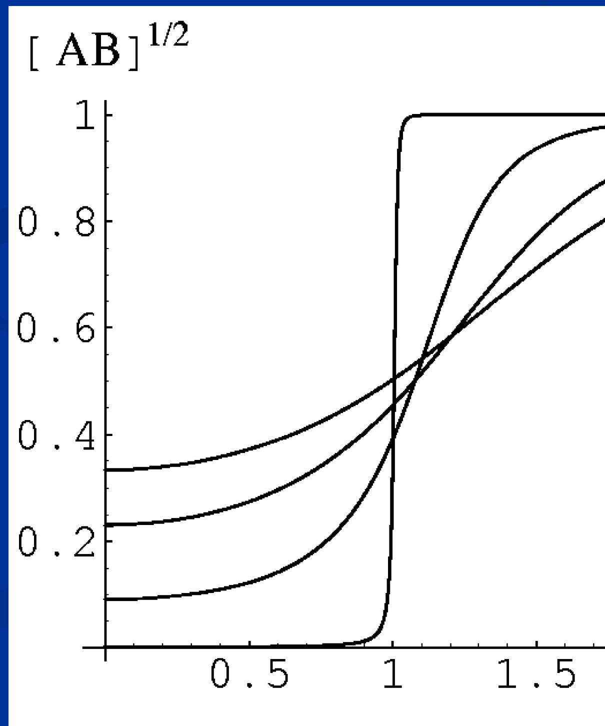


FIG. 2. A plot of $B(r)$ for $q=1$ and, reading from the top down, $c=0.5$, 0.3, 0.1, 0.001.



General approach

$$ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \qquad A = \frac{1}{V}$$

(i) V(r) attains minimum at $r^* \neq 0$ $\quad V(r^*) = \varepsilon \ll 1$

(ii) $\varepsilon \neq 0$ regular configuration

(iii) In limit $\varepsilon \to 0$ $V(r^*) \to 0$ $B(r) \to 0$ for all $r \leq r^*$

Consequences:

(a) infinite redshift

(b) infinite tidal forces for free-falling observer

Limit $\varepsilon \to 0$ Singular (degenerate) or regular?

Properties of spacetimes -?

Extremal RN outside – Minkowski inside (shell)

Classical model of electron
A. V. Vilenkin, P. I. Fomin 1978

Outer metric

$$ds^2 = -\left(1-\frac{m}{r}\right)^2 dt^2 + \left(1-\frac{m}{r}\right)^{-2} dr^2 + r^2 d\Omega^2 \qquad r \geq r_0$$

Inner metric

$$ds^2 = -\left(1-\frac{m}{r_0}\right)^2 dt^2 + dr^2 + r^2 d\Omega^2 \qquad r \leq r_0$$

Inside. Two alternatives

1) Using time *t as "good"* coordinate. Then $g_{00} \longrightarrow 0$

in the entire region $r \leq r_0$ Degenerate behavior

But Riemann tensor =0 there!

Surface $r = r_0$ becomes light-like in limit $r_0 \longrightarrow m + 0$

2) Let us introduce inside the coordinate *T* :

$$t = \frac{T r_0}{r_0 - m} \qquad ds^2 = -dT^2 + dr^2 + r^2 d\Omega^2$$

Finite intervals of *T* – infinite intervals of *t*

$$-dT^2 = -\frac{(r_0 - m)^2}{r^2{}_0} dt^2$$

Finite intervals of *t* – vanishing intervals of *T*

Let *T* be legitimate coordinate

$r_0 \rightarrow m$ Time-like surface. No matching between inside and outside

Complementarity

Zero mass particles

Radial motion $\lambda = \tilde{\varpi}^{-1} \int dl \sqrt{\tilde{B}}$

Outer region $\lambda - \lambda(r_0) \square \dfrac{r - m}{r_0 - m}$ infinite in the limit $r_0 \rightarrow m$

for any $r > m$

Boundary as impenetrable barrier

Naked behavior

$$R_{0r}{}^{0r} = K \qquad R_{0\theta}{}^{0\theta} = \bar{K} \qquad R_{\phi\theta}{}^{\phi\theta} = F \qquad R_{r\theta}{}^{r\theta} = \bar{F}$$

finite in limit $\sqrt{B(r_0)} \to 0$ everywhere in inner region

Kretschmann scalar is finite geometry is regular

free-falling frame Enhancement of curvature components

G. T. Horowitz and S. F. Ross PRD 1997, 1998

$$\bar{Z} = Z(2\frac{E^2}{B} - 1) \qquad Z = \bar{F} - \bar{K} \qquad \sqrt{B} \to 0 \quad Z \to \infty$$

$$Kr = R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}$$

Non-scalar polynomial curvature singularities
S. W. Hawking and G. F. Ellis, G. F. R. Ellis and B. G. Schmidt

Redshift

$$\omega(\infty) = \omega\sqrt{B(r)}$$ $B \to 0$ in the whole inner region

infinite redshift in quasi-horizon limit $\leftrightarrow$ impossibility to penetrate from inside to outside

End state of family of configurations

no way in which one can get a more compact object

no way to somehow turn it into extremal BH

Vacuum with surface layer: gluing between extremal Reissner-Nordstrom and Bertotti-Robinson metrics

Gluing between BR and extremal RN (O.Z., PRD 2004)

Another version of "classical electron"

$r \geq r_0 \quad B = (1 - \frac{m}{r})^2$

$r \leq r_0$ BR

$q = r_0$

Charge of shell

Surface stresses

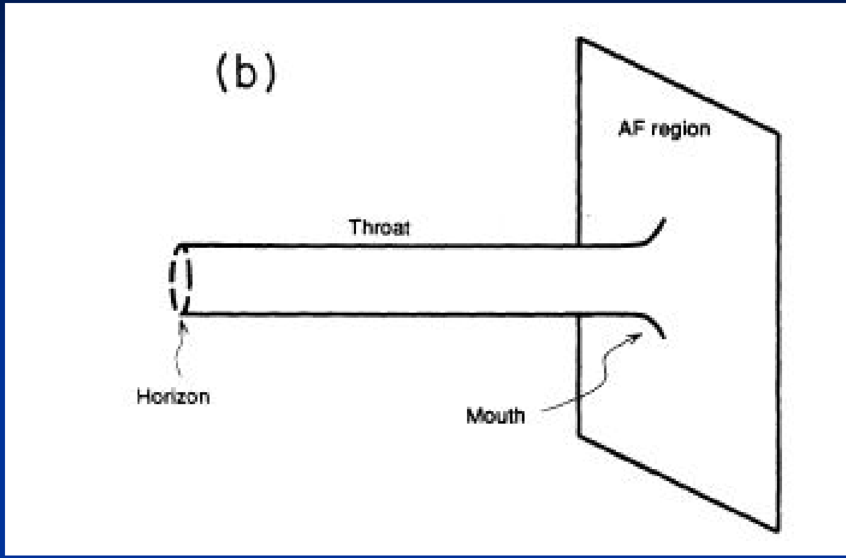
$$S_0{}^0 = \frac{\sqrt{B(r_0)}}{4\pi r^2{}_0} = \frac{\varepsilon}{4\pi r_0}$$

$$\varepsilon = r_0 - m$$

$$Q^{RN} - Q^{BR} = -\varepsilon$$

Contribution of shell vanishes in horizon limit, regular configuration

Extremal RN and BR



$$ds^2 = -dt^2 b^2 + dl^2 + r_0{}^2 d\omega^2$$

$$q = r_0$$

$$b = \sinh\frac{l}{r_0}, \exp\frac{l}{r_0}, \cosh\frac{l}{r_0}$$

For external observer like BH but no singularities inside

Reissner-Nordstrom core replaced by Bertotti-Robinson metric

Self-sustained configuration supported by electromagnetic forces without source
Mass without mass, charge without charge (Wheeler)

Reservation:
tidal stresses

$$\tilde{Z} = Z(\frac{2E^2}{B} - 1)$$

For BR Z=0

$$Y = S_1^{\ 1} - S_0^{\ 0} \qquad \tilde{Y} = \frac{1}{4\pi r^2_{\ 0}}(\varepsilon - \frac{2E^2 r^2_{\ 0}}{\varepsilon}) \sim \frac{1}{\varepsilon}$$

diverges: naked on surface

for $\varepsilon \neq 0$ boundary stresses finite and non-zero

$\varepsilon \to 0$ Stresses disappear in static frame, grow unbound in free-falling frame

Further property: regular QBHs (without infinite surface stresses) should be extremal

Boundary of body with Q<M cannot approach its own horizon: collapse

Particular models, numerics

De Felice et al CQG 1999

Analytical proof generalizing Buchdal limits for charged perfect fluid

Yunqiang and Siming 1999

General statement

Static regular configuration cannot approach its own horizon arbitrarily closely if horizon is non-extremal

Regular versus singular behavior and unattainability of quasi black hole limit

Kr finite but another manifestations of singular behavior

a) External observer: truly naked horizons and entire region in QBH limit
Boundary null

b) Internal observer: good inside but no matching to outside, boundary time-like

Complimentary relations between pictures viewed by different observers

Cannot arise from initially regular configuration, but approaches al closely as one likes (cf. 3d general law)

Usual horizon hides singularities, QH brings new singular features

Unusual counterpart of cosmic censorship

Inside may be regular and geodesically complete, no problem with limit

Mass of QBH

$$ds^2 = -N^2 dt^2 + dl^2 + g_{ab} dx^a dx^b$$

Size of body approaches gravitational radius

$$N \to 0$$

Mass formula for QBH vs. mass formula for BH

Key role of surface stresses

Tolman formula

$$M = \int d^3x \sqrt{-g}\,(T_k^{\ k} - T_0^{\ 0})$$

$$\sqrt{-g} = N\sqrt{g_3}$$

$$M_{tot} = M_{in} + M_{surf} + M_{out}$$

Inner mass

$$M_{in} \le N_{\max} M_{pr} const \to 0$$

Surface mass

$$M_{surf} = \int_{surf} d^3x\sqrt{-g_3}\,N(T_k^{\ k} - T_0^{\ 0}) \qquad T_\mu^{\ \nu} = S_\mu^{\ \nu}\delta(l)$$

$S_\mu^{\ \nu}$ Surface stresses in terms of jump of extrinsic curvature

$$M_{surf} = \frac{1}{4\pi}\int d\sigma[\left(\frac{dN}{dl}\right)_+ - \left(\frac{dN}{dl}\right)_-] \qquad S_a^{\ b} \square \delta_a^{\ b}\frac{N'}{N} \qquad N \to 0$$

$\left(\frac{dN}{dl}\right)_- \to 0$ since N bounded and $N \to 0$

$\left(\frac{dN}{dl}\right)_+ \to \kappa$ Surface gravity $M_{surf} \to \frac{\kappa A}{4\pi}$

Now, we allow infinite stresses and non-extremal QBH

Mass formula

$$M_{tot} = \frac{\kappa A}{4\pi} + M_{out} \qquad M_{out} = \varphi_h Q + M^{matter}_{out}$$

Smarr; Bardeen, Carter and Hawking

Now: different meaning

Role of stresses: $$S_a^{\ b} \sim \delta_a^{\ b} \frac{N'}{N}$$

Non-extremal case: streses diverge, contribution to mass finite

Extremal case: streses finite, contribution to mass vanishes

Hair properties: potential tends to constant,
charge distributions unaccessible from outside

Corolarry: Abraham-Lorentz electron in GR

Pure field model: 1) absence of bare non-EM stresses, 2) pure EM mass
Attempt: Vilenkin and Fomin 1978

Now seen: 1) is NOT equivalent to 2) in general

On quasihorizon of extremal QBH: stresses do NOT vanish but

their contribution to mass DOES vanish

Entropy of QBH

$r_B \to r_+$ $\qquad S \to \frac{A}{4}$ $\qquad$ Near formation of horizon

Time-like surface vs. light-like one

Einstein equations and thermodynamic (Jacobson 1995): Rindler observer near (but not exactly on) horizon

Limiting transition, continuity - ?

1) First law,
2) Temperature is equal to (or tends to) Hawking value

Assumptions:

1) First law,
2) Temperature is equal to (or tends to) Hawking value

Key role of surface stresses (no collapse – QBH!)

Unified approach:
Non-extremal, extremal: S=0 classically

Basic formulas

$$ds^2 = -N^2 dt^2 + dl^2 + g_{ab} dx^a dx^b$$

$$T = \frac{T_0}{N}$$

Surface gravity

$$Td(\sqrt{g}s) = d(\sqrt{g}\varepsilon) + \frac{\theta^{ab}}{2}\sqrt{g}dg_{ab}$$

$\theta^a{}_b$ stresses

$$g = \det g_{ab}$$

$$\theta_{ab} = \theta^g{}_{ab} - \theta^0{}_{ab}$$

$$8\pi\theta^g{}_{ab} = K_{ab} - Kg_{ab} + \frac{N'}{N}$$

K_{ab} Extrinsic curvature tensor

Non-extremal case

$$K_{ab} \sim l \qquad \theta_{ab} \sim N^{-1}N' \sim \frac{\kappa}{N}$$

$$d(\sqrt{g}s) \approx \frac{\kappa}{16\pi T_0}\sqrt{g}g^{ab}dg_{ab} \qquad T_0 \to \frac{\kappa}{2\pi}$$

$$d(\sqrt{g}s) = \frac{1}{4}d(\sqrt{g}) \qquad S = \frac{A}{4}$$

Bekenstein-Hawking value!

Role of stresses

$p + \rho = Ts$ Euler relation valid near horizon

$$p = \frac{1}{2}\theta_a{}^a \approx \frac{\kappa}{8\pi N} \to \infty$$

Extremal case

$$N \sim \exp(Bl) \qquad \frac{dN}{dl} \sim N \qquad p \sim N^{-1}N' \rightarrow const < \infty$$

$$d(\sqrt{g}s) = 0$$

In the quaishorizon limit

$$S = 0$$

Hawking, Horowitz, Ross
Gibbons and Kallosh
Teitelboim

classically

Mimickers of black holes

Possible alternative to black holes

$r \to r_+$ Looks almost like BH At infinity almost undistinguishable

Near-horizon region, strong gravity

Gravastars, wormholes, QBHs

$$8\pi S_2{}^2 \sim \frac{1}{\sqrt{-g_{00}}} \to \infty$$

Wormhole: no surface layer but $R \sim (-g_{00})^{-1}$

Lemos and O. Z. Spherically symmetric, Sushkov and O. Z. general static

$Lim(SO) \neq FO$

Summary

- Different types of objects: NEBH-EBH-QBH-star
- Unified approach to diverse systems (continuous and compact distributions of extreme dust, gravitating monopoles, glued vacua). Physically reasonable systems. Extremal charged dust: indifferent equilibrium
- External observer cannot distinguish gravitationally BH and QBH
- Their nature is different. Mutual impenetrability for QBH (one way penetrability for BH)
- BH: separation of causal nature. QBH: separation of dynamic nature. Rescaling of time or tidal forces (in bulk or on surface).
- Non-trivial combination of singular and regular features, complimentary relations depending on observer.
- Regular QBHs are extremal

$Lim(SO) \neq FO$

Summary

- If infinite stresses allowed: extend notion of QBH.
- Non-extremal: infinite stresses, finite non-zero contribution to mass. Extremal: finite stresses, vanishing contribution to mass. One-to-one correspondence with mass formula for BHs.
- Application to analogue of Abraham-Lorentz electron.
- Key role of infinite stresses in derivation of Bekenstein-Hawking entropy.
- Zero entropy of classical extremal QBH (finite stresses).
- Unified approach to limiting transition to BH state without forming horizon.
- Role of mimicker: singular behavior